STUDIES ON SAMPLING RELATIONS AND TIME-DELAY ESTIMATION IN FRACTIONAL FOURIER AND LINEAR CANONICAL TRANSFORM DOMAINS

by

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Dedicated to

My Parents ***Mr. R.K. Sharma, Mrs. Kastoori Devi***

Wife ***Sadhana,*** *Son* ***Avadhesh****, and Daughter* ***Ayushi,***

Sister-in-law ***Madhulata,*** *Nephew* ***Lokesh,*** *Nieces* ***Shobha, Deepti***

who are everything in my life.

CERTIFICATE

This is to certify that the thesis entitled " **Studies on Sampling Relations and Time-Delay Estimation in Fractional Fourier and Linear canonical Transform Domains"** being submitted by **Mr. Kamalesh Kumar Sharma** to the Indian Institute of Technology, Delhi, India for the award of the **Doctor of Philosophy** in Electrical Engineering is a bonafide research work carried out by him. He has worked under my guidance and supervision.

To the best of my knowledge the thesis has reached the requisite standard. The results presented in this thesis have not been submitted in part or full to any other university or institution for the award of any degree or diploma.

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ABSTRACT

Fractional Fourier transform (FRFT) and linear canonical transform (LCT) are generalizations of the conventional Fourier transform (CFT) and have received much attention in recent years. A number of useful properties of the FRFT/LCT have been derived which are generalizations of the corresponding properties of the CFT. The present thesis focuses on the investigation of two main issues related with the FRFT/LCT: (i) The sampling relations in the FRFT/LCT domains for different classes of signals, and (ii) The problem of time-delay estimation (TDE) using the FRFT.

Chapter 1 provides the motivation for the present work and gives an overview of the issues addressed in the Thesis. A brief review of the FRFT/LCT is also given here.

Chapter 2 presents the new sampling relations for bandlimited and bandpass signals in the FRFT/LCT domains. It is shown that for some specific classes of signals, perfect signal reconstruction from the signal samples is possible even if the sampling rate is less than the Nyquist rate. The idea of signal reconstruction from the signal samples in the FRFT/LCT domains using time-variant interpolating functions is introduced. A novel sampling scheme, which allows reduction in sampling rate for perfect signal reconstruction using linear time-variant (LTV) systems, is also presented. A generalized convolution theorem in the FRFT domains is introduced and generalized sampling expansions in the FRFT domains using the generalized convolution are derived. Sampling relations for the periodic and random signals in the FRFT/LCT domains are also presented.

Chapter 3 introduces several time-delay estimation (TDE) schemes using the FRFT in the stationary as well as nonstationary noise environments. Use of the FRFT in the statistical framework of TDE problem is discussed. The *maximum-likelihood* (ML) estimator for TDE using the fractional Fourier series (FRFS) is obtained, which generalizes the ML estimator based on the CFT. Lastly, the Cramer-Rao

lower bound (CRLB) in the FRFT domains as a generalization of the existing CRLB in the frequency domain is derived.

Chapter 4 discusses the extension of the TDE problem to 2-D cases. The image registration problems (involving either linear shifts or pure rotations in the given images to be registered) using the FRFT are investigated. An application of the TDE in a specific signal separation problem using the FRFT is also discussed and an iterative algorithm for signal separation is developed.

Chapter 5 investigates the issue of bandlimitedness of the signals in the LCT domains. The uncertainty relation in the LCT domains is also derived. The signal extrapolation by the method of alternating projections/POCS in the FRFT domains is presented. Some generalizations of the scaling property of the FRFT are also presented. The relationship amongst the FRFT and the chirp z-transform is studied and a new interpretation of the discrete approximation of the FRFT in terms of the samples of chirp z-transform on the unit circle is given.

Chapter 6 presents the conclusions of the present work and future scope of it.

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LIST OF ABBREVIATIONS AND SYMBOLS

AWGN	Additive White Gaussian Noise
AMDF	Average Magnitude Difference Function
ASDF	Average Squared Difference Function
AMSDF	Average Magnitude Squared Difference Function
AMSFDF	Average Magnitude Squared Fractional Difference Function
BPF	Band Pass Filter
CFT	Conventional Fourier Transform
CHT	Conventional Hilbert Transform
CFD	Conventional Fourier Domain
CFS	Conventional Fourier Series
CRLB	Cramer-Rao Lower Bound
DFRFT	Discrete Fractional Fourier Transform
DTFT	Discrete-Time Fourier Transform
DC	Direct Correlator
dB	Decibel

FRFTC	Fractional Fourier Transform Correlator
EM	Exponential Multiplication
FRFT	Fractional Fourier Transform
FRFS	Fractional Fourier Series
FIM	Fisher Information Matrix
FIR	Finite Impulse Response
GHT	Generalized Hilbert Transform
HPF	High Pass Filter
IR	Image Registration
IF	Instantaneous Frequency
IIR	Infinite Impulse Response
IS	Image Super-resolution
LCT	Linear Canonical Transform
LTI	Linear Time-Invariant
LTV	Linear Time-Variant
LPF	Low Pass Filter

ML	Maximum-Likelihood
MAP	Method of Alternating Projections
MAPFD	Method of Alternating Projections in the FRFT Domains
MSE	Mean Square Error
POCS	Projections onto Convex Sets
PDF	Probability Density Function
RHS	Right Hand Side
SNR	Signal-to-Noise Ratio
SFF	Self-Fourier Function
SFFF	Self-Fractional Fourier Function
TOA	Time of Arrival
TDOA	Time Difference of Arrival
TDE	Time-Delay Estimation
α	FRFT angle parameter
$F_\alpha(u)$	The FRFT of the signal $f(t)$
$\mathcal{F}_\alpha$	The forward FRFT operator with angle α

$\mathcal{F}_{\alpha}^{-1}$	The inverse FRFT operator with angle α
M	The Matrix of the LCT parameters $M=[A,\ B;\ C,\ D]$
$\Re_M$	The forward LCT operator
$F_M(u)$	The LCT of the signal $f(t)$
L_1	The Banach space of absolutely integrable functions
L_2	The Hilbert space of square integrable functions
$\Re_M^{-1}$	The inverse LCT operator for the parameter matrix $M=[A,\ B;\ C,\ D]$
$\tilde{\phi}_{\alpha,n}(u)$	Orthonormal basis of FRFS
$\mathrm{f}(t)$	A stochastic process
$*$	Conventional convolution operation
Θ	Generalized convolution operation
$f(t\theta\tau)$	Generalized translation of $f(t)$ by τ
$X^*(.)$	Complex conjugate of $X(.)$
T	Sampling time
ξ	Expectation operator

$\triangleq$	Equal by definition
Δu_{α}^{2}	Spread of the signal in the FRFT domain with angle α
Δu_{M}^{2}	Spread of the signal in the LCT domain with matrix M

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