

**MIXED/ EQUILIBRIUM/ HYBRID FINITE ELEMENT METHODS
FOR FOURTH ORDER ELLIPTIC PARTIAL DIFFERENTIAL
EQUATIONS WITH VARIABLE COEFFICIENTS**

by
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A Thesis submitted to the
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DEDICATED TO

Ramasamy, Thilakarajan, Rama

Meera, Muralidharan

mother, father

and


CERTIFICATE

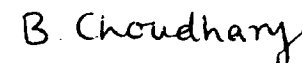
This is to certify that the thesis entitled:

'MIXED/EQUILIBRIUM/HYBRID FINITE ELEMENT METHODS
FOR FOURTH ORDER ELLIPTIC PARTIAL DIFFERENTIAL
EQUATIONS WITH VARIABLE COEFFICIENTS',

which is being submitted by Mr. S.Gopalsamy, Research Scholar, Mathematics Department to the Indian Institute of Technology, Delhi, for the award of the Degree of Doctor of Philosophy in Mathematics, is a record of bonafide research work carried out by him under our guidance and supervision and has fulfilled all the requirements for the submission of this thesis.

The results contained in this thesis have not been submitted in part or full, to any other University or Institute for the award of any degree or diploma.


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