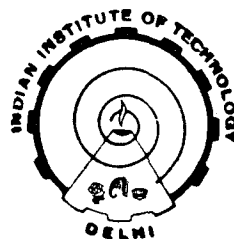


# **SEMI-CONTINUUM METHOD OF DESIGN OF SKEW GIRDER BRIDGES**

by

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*THESIS SUBMITTED  
IN FULFILMENT OF THE REQUIREMENTS OF  
THE DEGREE OF  
DOCTOR OF PHILOSOPHY*



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1988**

## CERTIFICATE

This is to certify that the thesis entitled "Semi-continuous Method of Design of Skew Girder Bridges" being submitted by Anant Ram Tiwari to the Indian Institute of Technology, Delhi for the award of Degree of Doctor of Philosophy in Civil Engineering is a record of bonafide research carried out by him. Shri Anant Ram Tiwari has worked under my guidance and supervision and fulfilled the requirements for the submission of the thesis which, to my knowledge, has reached the requisite standard.

The results contained in this thesis have not been submitted, in part or in full, to any other University or Institute for the award of any degree or diploma.



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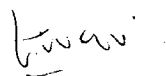
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Anant Ram Tiwari

## SYNOPSIS

Due to high traffic speeds, road or railway schemes can seldom be modified in order to eliminate the skew of their bridges, and therefore a considerable number of bridges are constructed with a greater or lesser degree of skew. With the growing demand for skew crossings in bridge constructions created by the continued growth and development of complex intersections and lack of space in congested areas, there is increasing tendency to adopt skew bridges. In mountaneous regions, some times a change in the alignment is not feasible for a right bridge and hence the provision of skew bridge construction is inevitable.

Design office methods for the rigorous and reliable analysis of these bridges with zero or very small angle of skew are readily available. The effect of skew above angles of about  $20^{\circ}$  in single span bridge decks can have a considerable effect on the behaviour of the bridge, especially in the short to medium range of spans where the span and width are of the same order.

Very limited literature is available on skew slab-on-girder bridges unlike skew slab bridges, where adequate amount of analytical and experimental investigations have been reported. No simple method of analysis is yet generally available with which a skew slab-on-girder bridge can be realistically analysed in the design office irrespective of the magnituide of its angle of skew. It is,

therefore, desirable to evolve a generalised design procedure for skew slab-on-girder bridges, after taking into account its actual structural behaviour with reasonable practical accuracy to suit the designers.

The proposed analysis is based on idealizing a bridge as semi-continuum which consists of an assembly of parallel longitudinal beams and an infinity of transverse beams. An elastic analysis of interconnected skew slab-on-girder bridges is presented in this thesis. The analysis is initially made for a three girder, simply supported skew bridge and then extended to the case of bridges having several longitudinals and to right bridges. Suggestions are also given for applicability of the method to continuous bridges.

Semi-continuum idealization is adopted wherein the longitudinal bending and torsional rigidities are concentrated in discrete longitudinal members and transverse bending and torsional rigidities are uniformly distributed along the length of the bridge. Only the first three harmonic terms of Fourier's sine series of applied loading are considered. Effect of the harmonic terms of applied loading higher than three are accurately taken into account assuming that these are fully retained by the respective loaded girders.

For each longitudinal, the final bending moment has been assumed to be made up of first three terms of Fourier's sine series. For each longitudinal, the rotational profile is made up of first harmonic of sine and first two harmonics of Fourier's cosine series. These then finally introduce six unknowns per girder, namely three relating to bending moment and the other three regarding torsional moment and rotation.

From the slope-deflection equations, the moments and shears per unit length of the transverse medium are formulated at the girder locations and the necessary linear simultaneous equations are derived. Six equations per girder formed are; 3 equations for girder bending moment, one equation for torsional moment and 2 equations for girder rotation relationship.

The solution of  $6N$  linear simultaneous equations for a  $N$ -girder bridge under applied harmonic loading give the bending and torsional moment coefficients along with the coefficient to be used in rotation. From these coefficients, other responses are obtained by successive integration or differentiation as the case may be.

The linear simultaneous equations for the skew bridge are in terms of four non-dimensional parameters and are solvable by computer for any type of loading and for bridge with any number of girders. The four dimensionless parameters appearing in the analysis are:

$\alpha$  Flexural parameter  
 $\mu$  Torsional parameter of longitudinal medium  
 $\lambda$  Torsional parameter of transverse medium  
 and  $k$  skew parameter of the bridge.

This thesis gives a generalised computer program for the analysis of N-girder skew bridge with these four non-dimensional parameters.

Through the analysis of three girder skew bridges, it is seen that the effect of  $\mu$  and  $\lambda$  could be combined and represented by a single dimensionless parameter for prediction of girder deflections. This is in conformity with the solution of orthotropic plates.

Denoting a single non-dimensional parameter  $\beta = \mu + \lambda$ ; the independent parameters are reduced to three, namely

$\alpha$  Flexural parameter  
 $\beta$  Torsional parameter  
 and  $k$  skew parameter of the bridge.

The ranges of variation of these parameters are discussed for practical three girder skew bridges. This combining is recommended in this thesis for preparation of design graphs for three girder bridges. For three girder skew bridges, the bending and torsional moment coefficients alongwith coefficient to be used in rotation have been obtained for various non-dimensional bridge parameters

covering the practical bridges for one harmonic loading at a time and a family of curves have been drawn for each coefficient.

Using these curves, the design bending moments, shear forces, longitudinal slopes, deflections, torsional moments and torsional rotations can be found out at desired locations on the girders. Thus the semi-continuum method is versatile in nature as the design of three girder skew bridges of any dimension and skewness upto  $k = 0.33$  is possible without using computer facility. A three girder skew bridge is analysed to illustrate the use of the design graphs.

For N-girder skew bridges, the presentation of design graphs will be quite cumbersome and also a reference to all of these might not be needed. Hence it is recommended that for skew bridges with more than three longitudinals, the solution should be from a small computer capable of solving linear simultaneous equations. However, the computer program given in chapter 8 of the thesis is capable of solving bridges upto 15 number of girders and from the solution various responses are computed at the desired locations along all the girders. There is an inbuilt facility in the program to account for the higher harmonic terms of applied loading in bending moments and shears. The girder rotations at the acute angled supports are also produced in the modified form.

Grillage Analogy method is a well established method of bridge analysis. To show the validity and accuracy of the semi-continuum method, two skew bridges having three and five girders are analysed by the presented method and alternately by Grillage Analogy method. The comparison of various girder responses under loading is made. It is found that the various responses always compare satisfactorily.

Some experimental and laboratory test results are available in the field of skew slab and beam bridge decks. For establishing the proposed method, three skew bridge models which were tested in the past and reported were analysed in this thesis by the semi-continuum method. The models are (i) a three girder  $45^\circ$  skew reinforced concrete bridge, (ii) a five girder  $45^\circ$  skew asbestos-cement bridge and (iii) a four girder  $30^\circ$  skew sand-Araldite bridge model.

The girder responses predicted by the semi-continuum method of analysis on skew bridge models are found comparable to those obtained by the laboratory tests on them. The semi-continuum method is very simple for skew girder bridge analysis as it requires a very small number of input data which comprises of few sectional and material properties of the bridge alongwith the loading details. The proposed method can thus be used with confidence for design of skew girder bridges.

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