

ACHIEVABLE RATE REGION FOR THE 2×2 MU-MISO OPTICAL BROADCAST CHANNEL WITH PER-LED POWER CONSTRAINTS

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DEPARTMENT OF ELECTRICAL ENGINEERING
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by

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Certificate

This is to certify that the thesis entitled “**Achievable Rate Region for the 2×2 MU-MISO Optical Broadcast Channel With Per-LED Power Constraints**” being submitted by **Mr. Amit Agarwal** to the department of Electrical Engineering, Indian Institute of Technology, Delhi (I.I.T. Delhi), for the award of the degree of **Doctor of Philosophy** is the record of the bona-fide research work carried out by him under my supervision. In my opinion, the thesis has reached the standards, fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted either in part or in full to any other university or institute for the award of any degree or diploma.

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Abstract

The increasing wireless data traffic, mostly generated inside office rooms and homes, is putting extra strain on the available radio frequency (RF) spectrum. This has motivated researchers to explore new frequency bands for wireless communications. One such viable option is optical spectrum. Wireless data transmission through optical spectrum (i.e., infrared (IR), visible light or ultraviolet) is known as optical wireless communication (OWC). In OWC systems, intensity modulation is widely employed as a transmission scheme, and direct detection is employed to recover the transmitted signal. The above mentioned scheme is known as intensity modulation with direct detection (IM/DD), and imposes a non-negativity constraint on the transmit signal (channel input). In addition to this, the channel input is peak power and average power constrained due to the practical and safety/lighting considerations. Due to these transmission constraints, the analysis done for RF channels is not directly applicable to the IM/DD channels. In this thesis, we focus on the study of an achievable rate region (ARR) of the zero-forcing (ZF) precoder in a multi-user multiple-input single-output (MU-MISO) IM/DD based optical broadcast channel (OBC) with channel state information available at the transmitter. Firstly, we consider the smallest instance of this channel, i.e. with 2 light emitting diodes (LEDs) at the transmitter and 2 users each having one photo-diode (PD). For this setting, we impose a per-LED peak power constraint along with a per-LED average power constraint with strict equality considered in visible light communication due to constant lighting requirement (referred as Type-II constraint). Next, we relax the average power constraint from strict equality to inequality, and study the effect of this relaxation on ARR of the 2×2 system under the stringent Type-II constraint. This relaxed form of Type-II constraint is referred as Type-I constraint. Type-I constraint is required in general OWC systems such as infra-red (IR) communication systems, where due to eye and skin safety regulations, the average power must be kept below a predetermined level. Lastly, we use the proposed theory of rate region characterization in a 2 LED 2 user scenario to derive the ARR in a N LED N user scenario. The thesis comprises three major chapters.

In the first chapter of the thesis, under Type-II constraint, we derive an ARR of the ZF precoder and mathematically characterize the boundary of the proposed ARR for a given channel realization, transmit optical signal-to noise ratio (SNR) and dimming level. Our study reveals that the boundary is Pareto-optimal, and instead of performing brute-force search over the entire rate region, this boundary characterization helps in computing many practical operating points. We also propose a novel transceiver architecture to operate at any point inside the rate region. At the transmitter, the dimming control and channel encoder are separated, which greatly simplifies the transceiver design. Using the proposed transceiver, our study reveals that the achievable information rates are sensitive to the placement of the LEDs/users. In a case study, for a fixed LEDs position, we compute optimum placement of the two users in a $(5 \times 5 \times 3) m^3$ office room setting. We also show that even with a substantial displacement of the two users from their optimum placement, the reduction in achievable rate is not significant. This study helps in defining the “coverage zones” for the users. We also study the behavior of the proposed ARR with the per-LED average optical power (dimming level) while keeping the peak power constant for a given transmit optical SNR and channel realization. Our study reveals that, the largest rate region is achieved when the per-LED average optical power is half of the allowed per-LED peak optical power.

In the second chapter of the thesis, we propose an achievable rate region of the ZF precoder and mathematically characterize the boundary under Type-I constraint. It is shown that for a fixed channel realization and transmit optical SNR, the ARR of the relaxed Type-I constrained system is the same as that of the stringent Type-II constrained systems, when the per-LED maximum allowed average optical power to peak power ratio $\beta \in [0, 1/2]$. Then we show that, when $\beta > 1/2$, the ARR under Type-I constraint remains constant with increasing β , whereas the ARR under Type-II constraint shrinks. Then, we propose a novel power efficient transmitter to operate at the boundary of the proposed ARR. Using this transmitter, it is shown that the total emitted optical power of a Type-I constrained system is significantly smaller than that of a Type-II system. Therefore, Type-I constrained systems are in-general more power and energy efficient than the stringent Type-II constrained systems. It is also shown that when operated at the boundary, at least one LED transmits at its full allowed

average optical power.

In the last chapter of the thesis, we consider the problem of obtaining achievable rate region of the ZF precoder in a $N \times N$ (when $N > 2$) MU-MISO IM/DD channel using the theory developed for 2×2 IM/DD channel. For 3 LED 3 user case we have numerically computed the boundary of the ARR under the both relaxed Type-I and stringent Type-II constraints. It is observed that the ARR under the both Type-I and Type-II constraints is same when $\beta \in [0, 1/2]$, and the largest rate region is achieved when $\beta = 1/2$. Under Type-II constraint, the ARR increases monotonically when $\beta \in [0, 1/2]$, and is symmetric about $\beta = 1/2$. When $\beta > 1/2$, ARR under Type-I constraint is the same as when $\beta = 1/2$. It is also observed that, when $\beta \in [0, 1/2]$ the total average emitted optical power is smaller for Type-I constrained systems, and at least one of the LEDs transmits at its maximum allowed average optical power.

सारांश

बढ़ते वायरलेस डेटा ट्रैफ़िक ज्यादातर ऑफिस रूम और घरों के अंदर उत्पन्न होते हैं और उपलब्ध रेडियो फ्रीक्वेंसी (आर एफ) स्पेक्ट्रम पर अतिरिक्त दबाव डाल रहा है। इस कारण वायरलेस संचार के लिए नए आवृत्ति बैंड का पता लगाने के लिए शोधकर्ता प्रेरित हुआ है। एक इस तरह के व्यवहार्य विकल्प ऑप्टिकल स्पेक्ट्रम है। ऑप्टिकल के माध्यम से वायरलेस डेटा ट्रांसमिशन (यानी, अवरक्त (आईआर), दृश्यमान प्रकाश या पराबैंगनी) को ऑप्टिकल वायरलेस के रूप में जाना जाता है। ऑप्टिकल वायरलेस संचार प्रणालियों में ट्रांसमिशन स्कीम के रूप में इंटेंसिटी मॉड्यूलेशन व्यापक रूप से कार्यरत है, और ट्रांसमिटेड सिग्नल को रिकवर करने के लिए डायरेक्ट डिटेक्शन उपयोग किया जाता है। इस योजना को आई एम / डी डी के रूप में जाना जाता है और यह ट्रांसमिट सिग्नल पर नॉन- नेगेटिव कंस्ट्रेंट अप्लाई करता है। इसके अतिरिक्त, चैनल इनपुट व्यावहारिक और सुरक्षा / प्रकाश व्यवस्था के कारण पीक पावर और औसत पावर कंस्ट्रेंट होता है। इन कंस्ट्रेंट के कारण आर एफ चैनलों के लिए किया गया विश्लेषण सीधे आई एम / डी डी चैनलों के लिए लागू नहीं होता है। इस थीसिस में हम ट्रांसमीटर पर चैनल की जानकारी होने पर बहु-उपयोगकर्ता मल्टीपल-इनपुट एकल-आउट में शून्य-फोर्सिंग के प्राप्य क्षेत्र के अध्ययन पर ध्यान केंद्रित करते हैं। सबसे पहले, हम सबसे छोटे उदाहरण अर्थात् ट्रांसमीटर पर २ प्रकाश उत्सर्जक डायोड (एल ई डी) और २ उपयोगकर्ता जिसमें प्रत्येक में एक फोटो-डायोड (पी डी) है पर विचार करते हैं। इस सेटिंग के लिए, हम प्रति एल ई डी पीक पावर के साथ दृश्य प्रकाश संचार (VLC) में इलुमिनेशन आवश्यकता के कारण एक प्रति-एलईडी फिक्स्ड औसत कंस्ट्रेंट लगाते हैं इसे टाइप -२ कंस्ट्रेंट नाम देते हैं। इसके बाद, हम फिक्स्ड औसत कंस्ट्रेंट को रिलैक्स करते हैं और २ X २ सिस्टम के प्राप्य क्षेत्र पर इस छूट का अध्ययन करते हैं। टाइप -२ कंस्ट्रेंट के रिलैक्स रूप को टाइप- १ नाम दिया गया है। अंत में, हम प्रस्तावित थ्योरी का उपयोग N - एल ई डी N - यूजर्स के प्राप्य क्षेत्र निकालने में करते हैं। थीसिस में तीन प्रमुख अध्याय शामिल हैं।

थीसिस के प्रथम अध्याय में हम टाइप-२ कंस्ट्रेंट के अंतर्गत शून्य-फोर्सिंग के प्राप्य क्षेत्र को निकालते हैं और किसी दिए गए चैनल, ऑप्टिकल सिग्नल-टू नॉइज़ अनुपात (एस एन आर) और डिमिंग स्तर के लिए प्राप्य क्षेत्र की सीमा को चिह्नित करते हैं। हमारे अध्ययन से पता चलता है कि सीमा पारेतो ऑप्टीमल है। हम एक नावेल ट्रांसीवर डिज़ाइन प्रोपोज करते हैं जिससे हम किसी भी ऑपरेटिंग पॉइंट जो की प्राप्य क्षेत्र के अंदर है पर काम कर सकते हैं। ट्रांसमीटर पर, डिमिंग कंट्रोल और चैनल एनकोडर अलग हो जाते हैं, जो ट्रांसीवर डिज़ाइन को बहुत सरल बनाते हैं। प्रस्तावित ट्रांसीवर का उपयोग करके हमारे अध्ययन से पता चलता है कि प्राप्य दर एल ई डी / उपयोगकर्ताओं की स्थिति के प्रति संवेदनशील हैं। एक केस स्टडी के अध्ययन में, ऑफिस रूम सेटिंग में फिक्स्ड एल ई डी स्थिति के लिए, हम दो उपयोगकर्ताओं का ऑप्टिमम प्लेसमेंट गणना करते हैं। यह भी पाया गया है कि दो उपयोगकर्ताओं को उनके ऑप्टिमम प्लेसमेंट से पर्याप्त विस्थापन के साथ प्राप्य दर में कमी ज्यादा नहीं है। यह अध्ययन उपयोगकर्ताओं के लिए कवरेज क्षेत्र परिभाषित करने में मदद करता है। हम प्रस्तावित प्राप्य क्षेत्र के व्यवहार का प्रति-एलईडी पीक पावर फिक्स रखते हुए, प्रति-एलईडी औसत ऑप्टिकल पावर (डिमिंग स्तर) के साथ अध्ययन करते हैं। हमारे अध्ययन से पता चलता है प्रति-एलईडी औसत ऑप्टिकल पावर प्रति एलईडी पीक ऑप्टिकल से आधी होने पर सबसे बड़ी दर क्षेत्र हासिल की जाती है।

थीसिस के दूसरे अध्याय में, टाइप-१ कंस्टेंट के अंतर्गत हम शून्य-फोर्सिंग के प्राप्य क्षेत्र का अध्ययन करते हैं और इसकी सीमा को निकालते हैं। फिक्स चैनल के लिए टाइप-२ और टाइप-१ सिस्टम्स का प्राप्य क्षेत्र एक जैसा होता है जब तक की प्रति एल इ डी औसत पावर और प्रति एल इ डी का रेश्यो जीरो से हाफ तक होता है। उसके बाद जैसे की यह रेश्यो हाफ से ज्यादा होता है तो टाइप-२ सिस्टम्स का प्राप्य क्षेत्र घटता है जबकि टाइप-१ का प्राप्य क्षेत्र कांस्टेंट हो जाता है। इसके बाद हम प्राप्य क्षेत्र की सीमा पर काम करने के लिए एक नावेल ट्रान्सीवर प्रोपोज करते हैं। इस ट्रान्सीमिटर का उपयोग करने से टाइप-१ सिस्टम्स की टोटल एमिटेड पावर टाइप-२ सिस्टम्स से काफी कम हो जाती है। इसलिए टाइप-१ सिस्टम्स टाइप-२ सिस्टम्स से ज्यादा पावर और एनर्जी एफिसिएंट है। यह भी पाया गया है की सीमा पर केवल एक ही एल इ डी अधिकतम पावर पर ऑपरेट करती है।

थीसिस के अंतिम अध्याय में, हम $N \times N$ सिनेरियो के लिए शून्य-फोर्सिंग के प्राप्य क्षेत्र का अध्ययन करते हैं। 3 एल इ डी 3 उपयोगकर्ता सिनेरियो के तहत टाइप-२ और टाइप-१ के लिए प्राप्य क्षेत्र की सीमा की गणना संख्यात्मक रूप से की गयी है। फिक्स चैनल के लिए टाइप-२ और टाइप-१ सिस्टम्स का प्राप्य क्षेत्र एक जैसा होता है जब तक की प्रति एल इ डी औसत पावर और प्रति एल इ डी का रेश्यो जीरो से हाफ तक होता है। जब यह रेश्यो हाफ होता है प्राप्य क्षेत्र अधिकतम होता है। टाइप-२ के तहत प्राप्य क्षेत्र सामान्य रूप से बढ़ता है जब तक की प्रति एल इ डी औसत पावर और प्रति एल इ डी का रेश्यो जीरो से हाफ तक होता है। प्राप्य क्षेत्र रेश्यो हाफ के अराउंड सीमेट्रिक होता है। यह भी पाया गया है जब तक की प्रति एल इ डी औसत पावर और प्रति एल इ डी का रेश्यो जीरो से हाफ तक होता है टाइप-१ सिस्टम्स के द्वारा ट्रान्समिट पावर कम होती है और सीमा पर केवल एक ही एल इ डी मैक्सिमम पावर पर ट्रान्समिट करती है।

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