STABILIZATION AND SYNCHRONIZATION OF NONLINEAR SYSTEMS IN CONTRACTION THEORY FRAMEWORK

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by

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Certificate

This is to certify that the thesis entitled **"Stabilization and Synchronization of Nonlinear Systems in Contraction Theory Framework"** being submitted by Bharat Bhushan Sharma to the Department of Electrical Engineering, Indian Institute of Technology Delhi, for the award of the degree of Doctor of Philosophy is the record of the bonafide research work carried out by him under my supervision. In my opinion, the thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted either in part or in full to any other university or institute for the award of any degree or diploma.

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Abstract

Controlling the behaviour of systems and making them behave in a coordinated manner have been a challenging problem in control engineering. Main motivation for the work presented in this thesis is to relook the established results in the field of control and synchronization in contraction theory framework and to explore their applications to different set of problems. Here, for stability analysis of systems, primarily contraction theory based analysis is adopted. This theory provides a basic framework to analyze the stability of nonlinear systems in an alternative way to the traditional Lyapunov based analysis. Along with that, this theory offers many significant advantages particularly in establishing exponential stability and exploring the synchronization of complex networks.

The work presented in the thesis addresses various aspects of controller & observer design for nonlinear systems especially chaotic systems and explores their applications in different domains. Contraction theory based systematic procedure is presented for stabilization of class of continuous time systems in strict feedback form. Proposed procedure is modified to address stability of systems with uncertain parameters. Further, recursive design of controller to address stability of discrete time strict feedback systems is proposed.

In next part of the thesis, two-system synchronization in drive-response configuration is analyzed. In this case, three different problems are addressed:

1. Synchronization of two given systems of same order (conventional synchronization) or different order (reduced order synchronization) is achieved by designing suitable control law using contraction theory approach. The procedure is based on error formulation and addresses synchronization for the cases with and without uncertainty in parameters.

2. Observer/estimator design is analyzed for the case of estimation of unknown frequencies of a sinusoidal signal. Here, signal is realized as an output of a state space dynamical system. An estimator is proposed for this system and its stability is analyzed using contraction theory.

3. Observer based synchronization scheme in transmitter-receiver configuration is presented using contraction theory and its application to secure communication is analyzed. For secure communication, one of the chaotic states is used to encrypt the information signal using n-shift ciphering algorithm. To improve the security, another chaotic state is taken as key for encryption.

With the background of synchronization of two systems, analysis is extended to address synchronization of networks of nonlinear systems in various configurations. To explore the proposed results, synchronization of simple chaotic systems and multi-scroll chaotic systems is addressed in different configurations. Analysis is made for the networks having systems in all-to-all and nearest neighbour coupling configurations. The synchronization results established in this case are global in nature.

Finally, stabilization of interconnected systems is addressed by using contraction theory based procedure. The individual members of the system have unstable linear or nonlinear dynamics. Moreover, coupling between these systems may be nonlinear. Design of decentralized controller based on systematic approach in contraction framework is proposed for such interconnected systems. Suitable interconnecting strengths or controller gains required for stabilization of overall system is achieved using Gerschgorin theorem. Application of the proposed results is explored for an interconnection of large number of mass-spring systems in a string. In this case, interconnections between the neighbouring systems are considered through springs exhibiting nonlinear characteristic. The efficacy of the various results presented in this thesis has been validated through simulation studies.

Contents