

AUTOMORPHISMS OF CERTAIN RELATIVELY FREE GROUPS AND ONE RELATOR GROUPS

By

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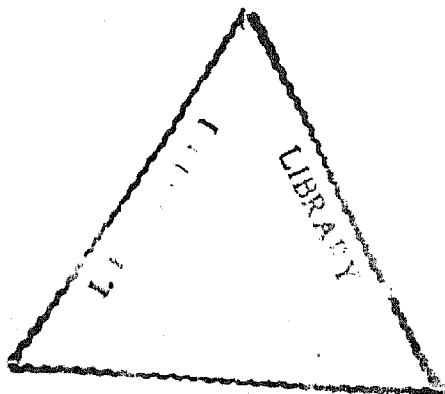
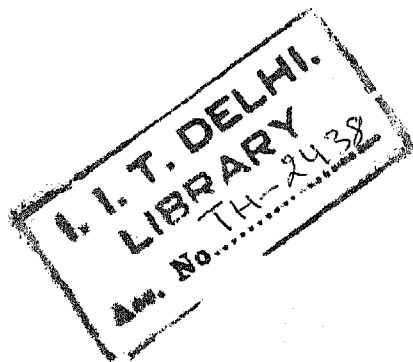
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To my beloved parents

V. Narayanaswamy

and

N. Mathuram

Certificate

This is to certify that the thesis entitled "Automorphisms of Certain Relatively Free Groups and One Relator Groups" which is being submitted by N. Chandramowliswaran for the award of DOCTOR OF PHILOSOPHY (MATHEMATICS) to the Indian Institute of Technology, Delhi, is a record of bonafide research work carried out by him under my guidance and supervision. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree. The results obtained in the thesis have not been submitted to any other University or Institute for the award of any degree or diploma.


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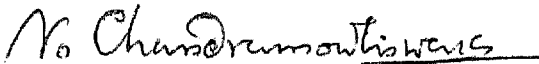
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N. Chandramowliswaran

Notations

$\subset$	Proper inclusion
$\subseteq$	Inclusion
$\mathbb{N}$	Set of natural numbers
$\mathbb{Z}$	Set of integers
$H \leq G$	H is a subgroup of G
$N \trianglelefteq G$	N is a normal subgroup of G
$Z(G)$	Center of G
x^y	$y^{-1}xy$
$[x, y]$	$x^{-1}y^{-1}xy$
$[x_1, x_2, \dots, x_n, x_{n+1}]$	$[[x_1, x_2, \dots, x_n], x_{n+1}]$
$\gamma_n(G)$	The n^{th} term of the lower central series of G
$\delta_n(G)$	the $(n+1)^{th}$ term of the derived series of G
$[H, K]$	The subgroup generated by all commutators $[h, k], \quad h \in H, k \in K$
G'	The commutator subgroup of G
F_n	The free group on $X_1, X_2, \dots, X_n$
F	The free group on $X_1, X_2, \dots, X_n, \dots$
$\mathbb{Z}G$	The integral Group ring of G
$\mathbb{Z}F_n$	The integral Group ring of the free group F_n
Δ_{F_n}	The augmentation ideal of $\mathbb{Z}F_n$
$GL_n(\mathbb{Z}F_n)$	The general linear group of degree n over $\mathbb{Z}F_n$
Δ_R	The left ideal of $\mathbb{Z}F_n$ generated by all elements of the form $\{r - 1 \mid r \in R, R \leq F_n\}$
$GL_n(R)$	The general linear group of degree n over the ring R
$GL_n(\mathbb{Z})$	The group of $n \times n$ matrices with integral entries whose determinant is ± 1

$Aut (G)$	Automorphism group of G
$Inn (G)$	The group of inner automorphisms of G
$IA (G)$	The kernel of the natural homomorphism $\{Aut (G) \longrightarrow Aut (G/G')\}$
$End (G)$	The semi-group of all endomorphisms of G
$\frac{\partial(W)}{\partial X_j} = \partial_j(W)$	The left Fox derivative of the freely reduced word $W \in F_n$ w.r.t X_j
$\frac{\partial'(W)}{\partial X_j} = \partial'_j(W)$	The right Fox derivative of the freely reduced word $W \in F_n$ w.r.t X_j

ABSTRACT

As the title of the thesis "Automorphisms of Certain Relatively Free Groups and One Relator Groups" suggests, our main aim in this thesis is to attempt and solve some problems on automorphism groups of some relatively free groups and one relator groups of special type. In chapter 1, we have discussed the lower central series, the derived series, residually finite groups, varieties of groups and relatively free groups. We have given a brief and up-to-date survey of known results relevant to our works in this thesis together with our comments and remarks. In chapter 2, we have obtained some results on group rings and applied free differential calculus, developed by Fox to get a new test word for determining certain automorphisms. We also discussed J.S. Birman's theorem and Krasnikov's theorem to get a relation between automorphisms and invertible matrices over certain integral group rings. In chapter 3, we have obtained several important results of the thesis related to automorphisms of certain relatively free groups. It contains several results which play a significant role in lifting automorphisms and non-tame automorphisms of certain groups.

We have derived a very explicit criterion for determining whether certain special types of endomorphisms of free groups of finite rank are automorphisms. We have also constructed a class of non-tame automorphisms of the groups $F_2/\gamma_m(R)$, $m \geq 3$, where R is a non trivial normal subgroup of free group F_2 .

Chapter 4 deals with automorphisms of two generator one relator groups. For certain two generator torsion-free one relator group G , it has been shown that the automorphism group $Aut(G)$ of G coincides with the group of IA-automorphisms of G . Several examples are given. In chapter 5, we have obtained a characterization of semi complete groups analogous to Baer's direct factor property criterion for complete groups. Many more results and several examples are given to demonstrate the situations where $IA(G) = Inn(G)$. At the end we have given some concluding remarks and recorded 9 problems which arose during our work.

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