

IDENTIFICATION, CLASSIFICATION AND ISOMORPHISM
OF
KINEMATIC CHAINS AND MECHANISMS

THESIS
SUBMITTED IN PARTIAL FULFILMENT OF THE
REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

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AUGUST, 1982

CERTIFICATE

Certified that the work reported in this thesis entitled, "IDENTIFICATION, CLASSIFICATION AND ISOMORPHISM OF KINEMATIC CHAINS AND MECHANISMS" has been done by Shri Vishnu Prakash Agrawal under my supervision, in partial fulfilment of the requirements for the degree of 'Doctor of Philosophy' in Mechanical Engineering, in the Department of Mechanical Engineering, Indian Institute of Technology, Delhi.

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ACKNOWLEDGEMENTS

I wish to express my deep sense of gratitude to Prof. J.S. Rao, D.Sc., Head, Mechanical Engineering Department, Indian Institute of Technology, Delhi, for his invaluable guidance and supervision and encouragement throughout the period of thesis work.

I sincerely express my thanks to Prof. A.C. Rao, D.Sc., Principal, Government Engineering College, Bilaspur (M.P.) for his valuable guidance and encouragement before and during the course of thesis work and to Dr. S.K. Gupta, Lecturer in the Centre for Computer Science and Engineering, I.I.T. Delhi, for useful discussions.

I also express my thanks to Prof. B.C. Nakra, Prof. K.N. Gupta, Prof. H.B. Mathur of Mechanical Engineering Department and Prof. C.V. Ramakrishnan and Prof. R. Natrajan of Applied Mechanics Department, I.I.T. Delhi for encouragement.

Lastly, I thank Mr. V.P. Gulati, Senior Stenographer, Mechanical Engineering Department for taking pains in typing the manuscript.

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ABSTRACT

An attempt has been made in this thesis to develop computationally simple and efficient analytical methods, associated with matrix methods and link-link variable characteristic polynomials (VCP-L), link-link variable permanent functions (VPF-L), loop-loop variable permanent functions (VPF-Lo) etc., for identification and isomorphism of kinematic chains (planer, spatial, simple-and multiple-jointed, with different types of joints), mechanisms, path generators and function generators; a complete matrix representation of kinematic chains resulting in the development of unique mathematical expressions called variable permanent functions/variable characteristic polynomials which identify kinematic chains upto isomorphism and leads to explicit expressions to determine characteristic coefficients analytically or by visual inspection of the chain. These analytical methods have been extended for identification and isomorphism of directed and undirected graphs in graph theory. Structural invariants of the kinematic chain contained in link-link characteristic polynomial (CP-L), VPF-L, VPF-Lo etc. and permanent of submatrices M_{ij} of path VPF-L, path VPF-Lo etc. have been identified and a simple method for computing them is developed. Different mobility properties of multi-loop

mechanisms have been identified and used for identification of pin-jointed kinematic chains topologically. Few analytical test procedures have been developed for determination of total, partial and fractionated degree-of-freedom of mechanisms and redundant links, if any, in function generators. All possible groups of kinematic chains with 3 to 5 loops have been identified and enumerated, and a scheme for structural classification of multi-loop kinematic chains based on loop connectivity properties has been proposed.

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