A THESIS ON

# SOME PROBLEMS IN FLUID DYNAMICS OF POROUS MEDIA AND RAREFIED GAS DYNAMICS

By

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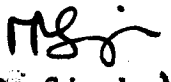
# C E R T I F I C A T E

This is to certify that the thesis entitled "Some Problems in Fluid Dynamics of Porous Media and Rarefied Gas Dynamics" which is being submitted by Mr. Prem Sagar Manocha for the award of Doctor of Philosophy (Mathematics) to the Indian Institute of Technology, Delhi, is a record of bonafide research work. He has worked for the last three years under my guidance and supervision.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.



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# A C K N O W L E D G E M E N T S

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# S Y N O P S I S

The present thesis incorporates some problems of fluid mechanics of permeable media and rarefied gas dynamics. The complete discussion has been carried out in four chapters. The first chapter introduces the flow through porous media. The second chapter deals with some problems of the flow past a permeable sphere. The third chapter gives the exposition of rarefied gas dynamics, and the fourth and last chapter deals with some problems of various simple geometries in rarefied gas dynamics.

## CHAPTER - I

This introductory chapter introduces the fundamental concepts, definitions and mathematical and physical aspects of the flow through porous media. In this chapter, some concepts of the theory of matched asymptotic expansions are also included.

## CHAPTER - II

The second chapter deals with the flow and heat transfer problems of a porous sphere at low Reynolds number. This chapter is divided into three sections. In the first section, the problem of steady heat transfer from a porous sphere in a stream of viscous incompressible fluid for the

of steady heat transfer of rarefied gas in plane Couette flow. In this problem, the linearized equation with BGK-model of the Boltzmann equation is considered and Shen's method is used in solving the problem. Numerical results are obtained for the heat flux vector and temperature jump at the plates, and these results show fairly good agreement with the previous studies.

In the second section, the problem of heat transfer between two concentric spheres with internal degree of freedom is studied. A diatomic gas is considered, and the method employing two sided Maxwellian type distribution as weighting function is used. After considering six moments, six unknown quantities (density, $n_1$, $n_2$; translational temperature, $T_{t1}$, $T_{t2}$; and internal temperature, $T_{i1}$, $T_{i2}$) are determined. Analytic solutions of the heat flux vector and translational and internal temperature profiles are obtained.

In the last section of the fourth chapter, the convergence of the discrete ordinate method and its application is discussed. The basic idea of this method consists of expressing the integral over velocity space of the perturbed distribution function in a linearized BGK-model or Ellipsoidal model of the Boltzmann equation, as finite summations according to Gauss-Hermite quadrature and

replacing the integrodifferential equations by a system of linear equations. In the first fold, the uniform convergence of different cases (e.g. plane Couette flow and Poisieulle flow, heat transfer in plane Couette flow), which have already been solved by this method, are established. In the second fold, the problem of heat transfer of rarefied gas in plane Couette flow using the Ellipsoidal model and employing the discrete ordinate method is discussed. Numerical results are obtained of the heat flux vector and temperature jump at two plates.

# C O N T E N T S