

**STUDY OF AERO-ELASTIC INTERFERENCE,
VORTEX INDUCED VIBRATION AND RELIABILITY:
IMPLICATIONS FOR THE DESIGN OF TALL
SLENDER STRUCTURES**

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Study of Aero-elastic Interference, Vortex Induced Vibration and Reliability: Implications for the Design of Tall Slender Structures

by

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Submitted

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CERTIFICATE

This is to certify that the thesis entitled “**Study of Aero-elastic Interference, Vortex Induced Vibration and Reliability: Implications for the Design of Tall Slender Structures**” being submitted by **Mr. Rishav** is the report of bonafide research work carried out by him under our supervision. This thesis has been prepared in conformity with the rules and regulations on Indian Institute of Technology, Delhi. We further certify that the thesis has attained a standard required for the award of **Doctor of Philosophy** degree of the Institute. The research reported and the results presented in the thesis have not been submitted, in part or full to any other institute or university for the award of any other degree or diploma.

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ABSTRACT

Tall slender structures such as chimneys are subjected to significant wind forces. In the present work, various aspects related to the design of such structures have been studied using experimental, numerical and analytical techniques. The phenomena studied include, vortex induced vibration, aerodynamic interference between geometrically similar structures and the effect of strakes. In addition, reliability assessment was carried out using fatigue and probability analysis.

The phenomenon of aerodynamic interference was studied across three cases: between two straight circular cylinders; a pair of 1:50 tapered cylinders; and a pair of 1:40 tapered cylinders—all in the staggered arrangement. Experiments were conducted in the 1.6m × 1.6m wind tunnel at IIT Delhi. The focus was on the enhanced across-wind response of the downstream cylinder near the first critical wind speed (at which the vortex shedding frequency matches the natural frequency of the structure). It is shown that the magnification in the response (with respect to that of the isolated cylinder) is a strong function of both the pitch and the orientation and extends to distances much larger than that reported in literature. It is evident from the current investigation that the magnification factors recommended in IS 4998 (Part 1: 1992, see Figure 1 therein/2015) and Eurocode 1 (Actions on structures- General actions- Part 1-4: Wind actions, see Equation E. 11 therein) need to be revised upwards. Further, the rule of thumb, used in the design of tall structures, that beyond 15 to 20 diameters interference is negligible is not tenable, especially for structures of identical geometry.

To study the effect of strakes on structures vibrating in second mode, a straight cylinder of aspect ratio 12 was chosen. Wind forces were applied in along wind and across wind

directions. To simulate periodic vortex shedding at lock-in, the across-wind force was modelled as sinusoidally varying with time at the second mode frequency. Its amplitude was allowed to vary with height in a manner consistent with a boundary-layer profile with a power-law exponent. Three cases have been compared— without strakes, strakes over the top one third, and strakes over the top half. It was found that the use of strakes has an adverse effect if the structure vibrates in the second mode. The effect is more severe if strakes are used over the top one-third in comparison to the top-half.

An alternate method for fluid-structure interaction (FSI) studies has been developed. The method is useful for 2-D problems and involves solving the governing equations in the frame of the oscillating structure. It is computationally more efficient and robust than the standard ALE (Arbitrary Lagrangian Eulerian) methods. In conjunction with an appropriate turbulence model (the SST $k-\omega$ model in this case) this method was able to numerically capture the lock-in phenomenon which is computationally very challenging.

A numerical model of a typical chimney was validated against experiments with a scaled down model. A comparative study of its response using different international codes for design of concrete chimneys showed that IS 4998 and CICIND are conservative while ACI 307-08 yields a more realistic response. The fatigue life of the chimney was estimated using wind loading based on the Emil Simiu spectra. The rainflow cycle counting method clubbed with the Palmgren-Miner rule was used in the analysis. To take into account the uncertainties in the system (like wind velocity, drag coefficient and material strength) probabilistic analysis was also done using the stochastic finite element method. It was found that the system is most sensitive to variations in wind speed.

सार

लम्बे पतले ढांचे (जैसे चिमनी) महत्वपूर्ण वायु बल के अधीन होते हैं। वर्तमान कार्य में, प्रयोगात्मक, संख्यात्मक और विश्लेषणात्मक तकनीकों का उपयोग करके ऐसे ढांचों के डिजाइन से संबंधित विभिन्न पहलुओं का अध्ययन किया गया है। इस शोध प्रबंध में भंवर प्रेरित कंपन, ज्यामितीय रूप से समान ढांचों के बीच वायुगतिकीय हस्तक्षेप और स्ट्रेक्स के प्रभाव का अध्ययन किया गया है। इसके अलावा, फटींग और संभाव्यता विश्लेषण का उपयोग करके विश्वसनीयता का मूल्यांकन किया गया।

वायुगतिकीय हस्तक्षेप की घटना का अध्ययन तीन मामलों में किया गया था: दो सीधे परिपत्र सिलेंडर के बीच; 1:50 टेपर सिलेंडरों की एक जोड़ी; और 1:40 टेपर सिलेंडर की एक जोड़ी - सभी स्टेजर व्यवस्था में। आईआईटी दिल्ली में 1.6 मीटर × 1.6 मीटर पवन सुरंग में प्रयोग किए गए। पहली महत्वपूर्ण पवन गति (जिस पर भ्रमिल अलगन आवृत्ति ढांचे की प्राकृतिक आवृत्ति से मेल खाती है) के पास डाउनस्ट्रीम सिलेंडर की बढ़ी हुई अक्रॉस-विंड प्रतिक्रिया पर ध्यान केंद्रित किया गया था। यह दिखाया गया है कि प्रतिक्रिया में बढ़ाई (पृथक सिलेंडर के संबंध में) पिच और कोण, दोनों का एक मजबूत फलन है और साहित्य में रिपोर्ट की गई तुलना में बहुत बड़ी दूरी तक फैलती है। वर्तमान जांच से यह स्पष्ट होता है कि आई. एस. 4998 (भाग 1: 1992, चित्र 1 उसमें / 2015) और यूरोकोड 1 (एक्शन्स ऑन स्ट्रक्चर्स - जनरल एक्शन्स- भाग 1-4: विंड एक्शन्स, समीकरण E. 11 देखें) की सिफारिश की गई आवर्धन गणक को संशोधित किए जाने की आवश्यकता है। इसके अलावा, सामान्य नियम जो लम्बे ढांचे के डिजाइन में उपयोग किया जाता है, कि 15 से 20 व्यास से अधिक वायुगतिकीय हस्तक्षेप नगण्य होता है, उचित नहीं है, विशेष रूप से समान ज्यामिति के ढांचों के लिए।

दूसरे मोड में कंपन करने वाले ढांचों पर स्ट्रेक्स के प्रभाव का अध्ययन करने के लिए, अभिमुखता अनुपात 12 का एक सीधा सिलेंडर चुना गया था। वायु बलों को हवा के साथ और अक्रॉस दिशाओं में लागू करा गया। लॉक-इन में आवधिक भ्रमिल अलगन को अनुकरण करने के लिए, अक्रॉस-विंड बल को समय के साथ दूसरी मोड आवृत्ति पर साइनोसॉइडली बदलकर प्रतिरूपण करा गया था। इसका आयाम ऊंचाई के साथ इस तरह बदला गया था कि यह पावर-लॉ प्रतिपादक वाली बाउंड्री-लेयर प्रोफ़ाइल के अनुरूप रहे। तीन मामलों की तुलना

की गई है- बिना स्ट्रेक्स के, शीर्ष पर एक तिहाई, और शीर्ष आधे पर स्ट्रेक्स। यह पाया गया कि ढांचे के दूसरे मोड में कंपन होने पर स्ट्रेक्स के उपयोग का प्रतिकूल प्रभाव पड़ता है। प्रभाव अधिक गंभीर होता है अगर शीर्ष-आधे की तुलना में शीर्ष एक-तिहाई पर स्ट्रेक्स का उपयोग किया जाता है।

पलूड-स्ट्रक्चर इंटरैक्शन (एफएसआई) अध्ययन के लिए एक वैकल्पिक विधि विकसित की गई है। विधि 2-डी समस्याओं के लिए उपयोगी है और इसमें दोलन ढांचे के फ्रेम में शासी समीकरणों को हल करना शामिल है। यह मानक ऐ.ल. ई (अर्बिट्ररी लैंग्रान्जियन आइलेरियन) विधियों की तुलना में संगणनात्मक रूप से अधिक कुशल और ठोस है। एक उपयुक्त टर्बुलेन्स मॉडल (इस मामले में एसएसटी के- ओमेगा मॉडल) के साथ संयोजन में यह विधि लॉक-इन घटना को संख्यात्मक रूप से प्रग्रहण करने में सक्षम रही जो संगणनात्मक रूप से बहुत चुनौतीपूर्ण होता है।

एक प्रारूपिक चिमनी के संख्यात्मक मॉडल को उसके एक स्केल डाउन मॉडल के साथ प्रयोगों का उपयोग करके मान्य किया गया था। कंक्रीट चिमनी के डिजाइन के लिए विभिन्न अंतरराष्ट्रीय कोड का उपयोग करके इसकी प्रतिक्रिया के तुलनात्मक अध्ययन से पता चला कि आई.एस. 4998 और सी.आई.सी.आई.एन.डी. रूढ़िवादी हैं जबकि ऐ.सी.आई. 307-08 अधिक यथार्थवादी प्रतिक्रिया देता है। एमिल सिमीउ स्पेक्ट्रा पर आधारित विंड लोडिंग का उपयोग करके चिमनी के फटींग जीवन का अनुमान लगाया गया था। विश्लेषण में पामग्रेन-माइनर नियम के साथ रेनफ्लो साइकिल गणना की पद्धति का उपयोग किया गया था। सिस्टम में अनिश्चितताओं को ध्यान में रखने के लिए (जैसे हवा का वेग, ड्रैग गुणक और भौतिक ताकत) स्टोकेस्टिक फाइनाइट एलिमेंट विधि का उपयोग करके संभाव्य विश्लेषण भी किया गया था। यह पाया गया कि सिस्टम हवा की गति में बदलाव के लिए सबसे अधिक संवेदनशील है।

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Nomenclature

\hat{a}	Unit direction vector of axis of rotation of non-inertial frame
a_0	Mass matrix multiplier
a_1	Stiffness matrix multiplier
A_l	Amplitude corresponding to t_l
A_n	Amplitude corresponding to t_n
a_L	Limiting deflection amplitude of chimney and is a fraction of D_l
$[b]$	Strain displacement matrix
B	Bandwidth of lift spectrum
BM	Bending moment
$[C]$	Damping matrix
C_D	Drag Coefficient
C_L	Lift Coefficient
C_p	Coefficient of pressure
C_{uu}	Auto-correlation function
$[d]$	Nodal displacement matrix
D	Outer diameter of structure. For tapered cylinders, it is diameter at 2/3 height
D_l	Average diameter of the top 1/3 rd section of the chimney

D_i	Damage index (n_i/N_i)
d_{rms}	rms of nodal displacement of numerical model of structure
D_T	Total damage ($\sum D_i$)
D_z	Outer diameter of chimney at height z
e	Average grit size of sand-cloth used to roughen the cylinders
E	Energy enhancement
$[E_y]$	Young's modulus matrix
f	Frequency
F	External body force
F_{11}	Power spectral density function
$F_{acr}(z)$	Force per unit length in across wind direction
f_c	Compressive strength of the concrete in MPa
f_d	Damped natural frequency
F_D	Along wind force (Drag force)
f_l	Frequency corresponding to energy containing scales ($\frac{U_{crit}}{2\pi l}$)
f_M	Monin coordinate
f_n	Natural frequency in cps (1/s)
f_s	Shedding frequency

f_{sr}	limit of inertial subrange (~ 0.2)
g	Limit state function
	$g(z) = 0$ limit state surface
	$g(z) < 0$ safe region
	$g(z) > 0$ failure region
G	Gap between two cylinder walls
G/D	Gap ratio
H	Height of cylinders
H/D	Aspect ratio
I	Turbulence intensity ($=u'/U$)
I_{NA}	Moment of inertia about neutral axis
J	Jensen number
k	wave number/frequency
$[K]$	Stiffness matrix
K_a	Aerodynamic damping parameter
k_p	Peak factor to calculate $F_{acr}(z)$
l	Integral length scale of the turbulence
L	Longitudinal spacing between cylinders (centre to centre)
M	Magnification factor

$[M]$	Mass matrix
M_b	Bending moment
m_i	Average mass per unit height (kg/m)
M_n	Modal mass
m_o	Equivalent mass per unit length
$m(z)$	Mass per unit length at height z
n	Number of cycles / number of data points
N	Total number (of cycles/random variables/samples)
N_c	number of cycles for concrete before failure in fatigue loading (fatigue life)
N_e	Number of elements
N_f	Number of samples
n_i	Number of cycles applied/happened at stress range S_i
N_i	Number of cycles for failure at stress range S_i
$[N_s]$	Shape factor matrix
p	Fluid pressure
P	Pitch i.e. centre to centre distance between two cylinders
P/D	Pitch ratio
P_f	Probability of failure
R	Resistance of a system (e.g. permissible strength or strain)

r_0	Position vector locating the origin of non-inertial frame
\bar{r}	Position vector of any point in the CFD domain with respect to origin of the moving frame
Re	Reynolds number
R_e	Reliability ($1 - P_f$)
R_l	Reynolds number based on integral scale ($u' l / \nu$)
rms	Root mean square
S	System response (e.g. strain or stress developed in a material)
Sc	Scruton number ($= 4\pi \xi_s m_o / \rho D_1^2$)
sd	Downstream distance from centre of cylinder (in terms of D)
SE_{μ_x}	Standard error of mean
S_i	Stress amplitude in stress cycles
S_m	Source term
Ss	Side-wise width of the domain (in terms of D)
St	Strouhal number, (fD/U)
Su	Upstream distance from centre of cylinder (in terms of D)
$S(z,f)$	Spectra for longitudinal velocity fluctuations at height z
t_s	Thickness
t	Time (s)

t_1	Time at peak of 1 st cycle(s)
T	Transverse spacing between cylinders (centre to centre)
T_d	Time required for a perturbation to travel down to dissipation scale
t_n	time at peak of n th cycle(s)
U	Wind velocity
U_l	Local mean velocity
U_{crit}	Velocity at which lock-in peak occurs ($f_n = f_s$)
U_H	Wind velocity at the top of model i.e. at height H
u_i	Standard normal variate
$\overline{u_r}$	Velocity of moving frame with respect to stationary frame
$u(t)$	Instantaneous velocity fluctuation
U_z	Wind velocity at height z
U_∞	Free stream wind velocity
u'	rms of fluctuating component of velocity
u^*	Shear or friction velocity of turbulent flow
$\overline{V_r}$	Relative velocity
V_t	Linear velocity of non-inertial frame with respect to stationary reference frame
x	Direction of wind

X, Y	Random variables contributing to strength/resistance and stress of a structure respectively
x_i	random strain data point (sample)
y	Across wind direction
y_c	Deflection of chimney
Y_r	Distance from neutral axis at which stress is to be determined
z	Distance from ground in height-wise direction
Z	Chimney's critical random variable (at 21m height); $Z(X, Y) = R(X) - S(Y)$
z_o	Aerodynamic roughness height
α	Power law coefficient for boundary layer profile
α_i	probabilistic sensitivity factor
β	reliability index (μ/σ)
θ	Incidence angle i.e. angle between pitch line (line joining center to center) and wind direction
θ_c	Critical incidence angle (angle for which magnification factor is highest, for respective pitch ratio)
κ	von Karman constant (= 0.4)
ν	Kinematic viscosity of air
ζ	Damping ratio

ξ_a	Negative aerodynamic damping ratio
ξ_s	Structural damping ratio
ζ	Mode shape
τ	Stress tensor
δ	Logarithmic decrement
λ	Load correction length
ϕ_i	Phase difference between different frequencies
μ_x	sample mean of x_i
μ	Mean
σ	standard deviation [of the sample/g(Z)]
σ_b	Bending stress
σ_c	Static flexural strength of concrete as defined by IS 456 (= $0.7\sqrt{f_c}$)
σ_{max}	Maximum stress in concrete
$[\sigma_s]$	Stress matrix
ρ	density of air
δ	Logarithmic decrement
$\bar{\omega}$	Angular velocity of rotation of non-inertial frame with respect to stationary reference frame
ω_n	Natural frequency (= $2\pi f_n$) in rps

ω_d	Damped natural frequency ($=2\pi f_d$) in rps
ω_z	Vorticity (z-component)
$[\partial]$	Gradient matrix in 3-D space and time
$[\varepsilon]$	Strain matrix
$\frac{\partial p}{\partial \mu}$	sensitivity of P_f with respect to μ
$\frac{\partial p}{\partial \sigma}$	sensitivity of P_f with respect to σ

Abbreviations

ALE	Arbitrary Lagrangian-Eulerian method
amp	Amplitude
avg	Average
BM	Bending moment
CDF	Cumulative distribution function
CFD	Computational Fluid Dynamics
CFL	Courant–Friedrichs–Lewy number
FE	Finite Element
FEA	Finite Element Analysis
FEM	Finite Element Method

FFT	Fast fourier transform
FORM	First order reliability methods
FRP	Fibre reinforced polymer
FSI	Fluid Solid Interaction
GPRSM	Gaussian Process Response Surface Method
HDPE	High-density polyethylene
ID	Internal diameter
IS	Indian Standard (Code)
<i>M</i>	Magnification factor
MCS	Monte Carlo simulation
NI	National Instruments
OD	Outer diameter
PDF	Probability density function
PIV	Particle image velocimetry
PSDF	Power spectral density function
rms	Root mean square
SFEM	Stochastic Finite Element Method
URANS	Unsteady Reynolds Averaged Navier-Stokes equations
VIV	Vortex Induced Vibration