

LINEAR STABILITY OF CONTAMINATED FLOWS

FAROOQ AHMAD BHAT



**DEPARTMENT OF APPLIED MECHANICS
INDIAN INSTITUTE OF TECHNOLOGY DELHI
OCTOBER 2020**

© Indian Institute of Technology Delhi (IITD), New Delhi, 2020

LINEAR STABILITY OF CONTAMINATED FLOWS

by

FAROOQ AHMAD BHAT

Department of Applied Mechanics

Submitted

in fulfilment of the requirements of the degree of Doctor of Philosophy

to the



INDIAN INSTITUTE OF TECHNOLOGY DELHI

OCTOBER 2020

Certificate

This is to certify that the thesis entitled, "**Linear Stability of Contaminated Flows**" submitted by **Mr. Farooq Ahmad Bhat** for the award of degree of Doctor of Philosophy to the Indian Institute of Technology Delhi is a record of bona fide research work carried out by him under my guidance and supervision.

Mr. Farooq Ahmad Bhat has fulfilled all the prescribed requirements and the thesis is, in my opinion, worthy of consideration for the degree of Doctor of Philosophy in accordance with the regulations of the Institute. The contents of this thesis have not been submitted in part or in full to any other university or institute for the award of any degree or diploma.

Dr. Arghya Samanta

Assistant Professor

Department of Applied Mechanics
Indian Institute of Technology Delhi
New Delhi-110016
India

Place: New Delhi

Date :

Acknowledgements

Ph.D is not a work and achievement of one individual, but of a collective enterprise of faculty, family, friends and community. It would be unfair to present this work without acknowledging the immense support of different people that I have received and made this thesis possible. First of all, thanks to "The ALMIGHTY ALLAH", with his gracious will, I was not only able to pursue this work but also complete it properly.

This work would not have been possible without the generous and inspiring support of my supervisor Dr. Arghya Samanta. His knowledge, wisdom and dedication towards his work has always inspired me. He has treated me, not just as student, rather as a friend and always advised me in most positive and constructive spirit. I will always be in debt to him for making me who I am today.

Beside my supervisor, I would also like to thank my student research committee members – Prof. S. Sanghi , Prof. S.V. Veeravalli, and Prof. S Datta for their valuable suggestions and comments throughout the research work.

I am also thankful to Prof. Sawan S. Sinha for his tremendous encouragement and words of wisdom. I would also like to acknowledge the Department of Science, Ministry of Human Resource Development for providing the financial assistance during this work.

I have been very fortunate to have very supportive friend like Dr. Murtaza Rasool, who not only guided me to join PhD at the first place but also helped

me in every possible way during the course of my stay in the institute. I am also thankful to Sagar Saroha and Nishant Parashar for making my stay in lab an enjoyable and educational time. I am also grateful to my dear friends Suhail Bashir, Harshvardhan Varma, Abhilasha Awasthi and Komal Tomar. Special thanks to Digpriya and Abhishek Rai for being with me all times.

Finally, I owe my genuine gratitude and honest appreciation to my parents and my family for their passionate help, blessings, and inspiration throughout the work. They supported me at each progression by contributing their significant time and never questioned on my capacity to finish the assignment. I required it the most and I will always be thankful to them.

Farooq Ahmad Bhat

Abstract

Falling films are widely encountered in nature, technical, and industrial setups, and the dynamics of waves generated on film surfaces play a major role in liquid-liquid extractor, multi-layer liquid coating, thin film heat exchangers, and chemical reactors. For this reason, it is not surprising that the studies of single or multi-layer falling films are of increasing interest. In this thesis, an attempt has been made to investigate the linear stability of single-layer and two-layer fluid flowing down an inclined plane when either fluid surface or both fluid surface and fluid-fluid interface are covered by insoluble surfactants.

First part of the thesis deals with the linear stability analysis of a fluid flowing down a slippery inclined plane when the free surface of the fluid is contaminated by a mono-layer of insoluble surfactant. The Orr-Sommerfeld equation (OSE) is derived for infinitesimal disturbances of arbitrary wave numbers. At low Reynolds number, the OSE is solved analytically by using the long-wave analysis, which shows that the critical Reynolds number decreases in the presence of a slippery plane but increases in the presence of an insoluble surfactant. This fact ensures a destabilizing effect of wall slip and a stabilizing effect of insoluble surfactant on the long-wave surface mode. Further, the Chebyshev spectral collocation method is implemented to tackle the OSE numerically for an arbitrary value of the Reynolds number, or equivalently, for an arbitrary value of the wave number. At moderate Reynolds number, wall slip exhibits a stabilizing effect on the surface mode as opposed to the result in the long-wave

regime, while the insoluble surfactant exhibits a stabilizing effect on the surface mode as in the result of the long-wave regime. On the other hand, at high Reynolds number, both wall slip and insoluble surfactant exhibit a stabilizing effect on the shear mode. Further, it is shown that both surface and shear modes compete with each other to dominate the primary instability once the inclination angle is sufficiently small. In addition, new phase boundaries are identified to differentiate the regimes of surface and shear modes.

The second part of the thesis deals with the study of linear stability analysis of a shear-imposed fluid flowing down an inclined plane when the free surface of the fluid is covered by an insoluble surfactant. The Orr-Sommerfeld boundary value problem is formulated and solved numerically based on the Chebyshev spectral collocation method. Two temporal modes, the so-called surface mode and surfactant mode, are detected in the long-wave regime. The surfactant mode becomes unstable when the Péclet number exceeds its critical value. In fact, the instability of the surfactant mode occurs on account for the imposed shear stress. Energy budget analysis predicts that the kinetic energy of the infinitesimal disturbance grows with the imposed shear stress. On the other hand, the numerical results reveal that both surface and surfactant modes can be destabilized by increasing the value of the imposed shear stress. Similarly, it is demonstrated that the shear mode becomes more unstable in the presence of imposed shear stress. However, it can be stabilized by incorporating the insoluble surfactant at the free surface. Apparently, it seems that inertia does not play any role in the surfactant mode in the moderate Reynolds number regime. Furthermore, the competition between surface and shear modes is discussed.

Finally, the third part of the thesis deals with a detailed study of linear stability analysis of a surfactant-laden two-layer film flowing down a slippery inclined

plane in the presence of several flow parameters. Four modes, the so-called surface mode, interface mode, surface surfactant mode and interface surfactant mode, are identified in the long-wave regime. It is found that the surface surfactant mode is always stable, but the interface surfactant mode can be unstable if the Péclet number Pe_2 corresponding to the interfacial surfactant exceeds its critical value and $mr > 1$, where m and r respectively stand for the viscosity ratio and the density ratio. Further, in the long-wave regime, the interface mode can be stabilized, but the surface mode can be destabilized by introducing the effect of wall slip when $m < 1$. However, the effect of wall slip on the interface and surface modes is completely opposite as soon as $m > 1$. Furthermore, the viscosity ratio provides a dual role in the primary instability generated by the surface mode, i.e., it exhibits a stabilizing effect close to the criticality but exhibits a destabilizing effect far away from the criticality. However, the above results regarding the surface mode are fully converse if the density ratio, or, the thickness ratio varies rather than the viscosity ratio. Moreover, the interface surfactant mode can be stabilized by increasing the magnitude of density ratio, viscosity ratio and thickness ratio. In addition, the shear modes appear in the numerical simulation when the Reynolds number is very large and the inclination angle is very small. The shear mode associated with the lower fluid layer can be stabilized, but the shear mode associated with the upper fluid layer can be destabilized by increasing value of the viscosity ratio.

सारांश

गिरने वाली फिल्मों का प्रकृति, तकनीकी और औद्योगिक सेट-अप में व्यापक रूप से सामना किया जाता है, और फिल्म सतहों पर उत्पन्न तरंगों की गतिशीलता तरल-तरल एक्सट्रैक्टर, बहु-परत तरल कोटिंग, पतली फिल्म हीट एक्सचेंजर्स और रासायनिक रिएक्टरों में एक प्रमुख भूमिका निभाती है। इस कारण से, यह आश्चर्यजनक नहीं है कि एकल या बहु-परत गिरने वाली फिल्मों के अध्ययन में रुचि बढ़ रही है। इस थीसिस में, झुकाव वाले सतह से नीचे बहने वाले एकल-परत और दो-परत के तरल पदार्थ की रैखिक स्थिरता की जांच करने का प्रयास किया गया है, जब या तो द्रव की सतह या तरल पदार्थ की सतह और द्रव-द्रव इंटरफेस दोनों अघुलनशील पृष्ठसक्रियकारक से आवृत किए जाते हैं।

थीसिस का पहला हिस्सा एक फिसलन झुकाव वाले सतह के नीचे बहने वाले तरल पदार्थ के रैखिक स्थिरता विश्लेषण से संबंधित है जब द्रव की मुक्त सतह अघुलनशील सर्फैक्टेंट की मोनो-परत द्वारा दूषित होती है। ऑर-सोमरफेल्ड समीकरण (OSE) विवेकाधीन तरंग संख्याओं के अतिसूक्ष्म विक्षेपों के लिए व्युत्पन्न है। कम रेनॉल्ड्स संख्या में, OSE को लंबी-तरंग विश्लेषण का उपयोग करके विश्लेषणात्मक रूप से हल किया जाता है, जिससे पता चलता है कि न्यूनतम सीमा रेनॉल्ड्स संख्या, फिसलन सतह की उपस्थिति में कम हो जाती है लेकिन एक अघुलनशील पृष्ठसक्रियकारक की उपस्थिति में बढ़ जाती है। यह तथ्य दीवार की फिसलन के अस्थिर प्रभाव और लंबी-तरंग की सतह मोड पर अघुलनशील पृष्ठसक्रियकारक के स्थिर प्रभाव को सुनिश्चित करता है। इसके अलावा, चैबीशेव वर्णक्रमीय विखंडन विधि रेनॉल्ड्स संख्या के एक विवेकाधीन मूल्य के लिए, या समकक्ष, तरंग संख्या के विवेकाधीन मूल्य के लिए संख्यात्मक रूप से OSE से निपटने के लिए लागू की जाती है। मध्यम रेनॉल्ड्स संख्या में, दीवार फिसलन सतह मोड पर एक स्थिर प्रभाव दिखाती है, जो लंबी- तरंग परिणामों के विपरीत है, जबकि अघुलनशील पृष्ठसक्रियकारक सतह मोड पर एक स्थिर प्रभाव प्रदर्शित करता है जैसा कि लंबी- तरंग परिणाम में होता है। दूसरी ओर, उच्च रेनॉल्ड्स संख्या में, दीवार फिसलन और अघुलनशील पृष्ठसक्रियकारक दोनों ही अपरूपण मोड पर एक स्थिर प्रभाव प्रदर्शित करते हैं। इसके अलावा, यह दिखाया गया है कि झुकाव कोण पर्याप्त रूप से छोटा होने पर प्राथमिक अस्थिरता पर हावी होने के लिए सतह और अपरूपण मोड दोनों एक दूसरे के साथ प्रतिस्पर्धा करते हैं। इसके अलावा, सतह और अपरूपण मोड के क्षेत्र को अलग करने के लिए नए अवस्था सीमाओं की पहचान की जाती है।

थीसिस का दूसरा भाग एक झुकाव वाले सतह से नीचे बहने वाले तरल पदार्थ के रैखिक स्थिरता विश्लेषण के अध्ययन से संबंधित है, जब द्रव की मुक्त सतह एक अघुलनशील पृष्ठसक्रियकारक द्वारा आवृत की जाती है। ऑर-सोमरफेल्ड सीमा मूल्य समस्या, चैबीशेव वर्णक्रमीय विधि के आधार पर संख्यात्मक रूप से तैयार और हल की गई है। दो सामयिक मोड, तथाकथित सतह मोड और पृष्ठसक्रियकारक मोड, लंबी-तरंग क्षेत्र में पाए जाते हैं। पृष्ठसक्रियकारक मोड अस्थिर हो जाता है जब पेक्लेट संख्या अपने न्यूनतम सीमा मूल्य से अधिक हो जाती है। वास्तव में, पृष्ठसक्रियकारक मोड की अस्थिरता लगाए गए अपरूपण तनाव के कारण होती है। ऊर्जा बजट विश्लेषण भविष्यवाणी करता है कि अतिसूक्ष्म विकेषों की गतिज ऊर्जा लगाए गए अपरूपण तनाव के साथ बढ़ती है। दूसरी ओर, संख्यात्मक परिणामों से पता चलता है कि सतह और पृष्ठसक्रियकारक मोड दोनों को लगाए गए अपरूपण तनाव के मूल्य को बढ़ाकर अस्थिर किया जा सकता है। इसी तरह, यह प्रदर्शित किया जाता है कि अपरूपण मोड लगाए गए तनाव की उपस्थिति में अधिक अस्थिर हो जाता है। हालांकि, इसे मुक्त सतह पर अपरूपण अघुलनशील पृष्ठसक्रियकारक को शामिल करके स्थिर किया जा सकता है। जाहिर है, ऐसा लगता है कि माध्यिम रेनॉल्ड्स संख्या क्षेत्र में जड़ता पृष्ठसक्रियकारक मोड में कोई भूमिका नहीं निभाती है। इसके अलावा, सतह और अपरूपण मोड के बीच प्रतिस्पर्धा पर चर्चा की गयी है।

अंत में, थीसिस का तीसरा हिस्सा कई प्रवाह के मापदंडों की उपस्थिति में फिसलन झुकाव वाले सतह से निच्ये बहते हुवे एक पृष्ठसक्रियकारक से लदे हुवे दो-परत फिल्म के रैखिक स्थिरता विश्लेषण के विस्तृत अध्ययन के दिखाता है । चार मोड, तथाकथित सतह मोड, इंटरफेस मोड, सतह पृष्ठसक्रियकारक मोड और इंटरफेस पृष्ठसक्रियकारक मोड, को लंबी-तरंग क्षेत्र में पहचाना गया है। यह पाया जाता है कि सतह पृष्ठसक्रियकारक मोड हमेशा स्थिर होता है, लेकिन इंटरफेस पृष्ठसक्रियकारक मोड अस्थिर हो सकता है अगर पेक्लेट नंबर Pe_2 , जो इंटरफेसियल पृष्ठसक्रियकारक से संबंधित है, अपने न्यूनतम सीमा मूल्य से अधिक हो और $mr > 1$, जहां क्रमशः m और r , श्यानता अनुपात और घनत्व अनुपात के प्रदर्शित करते हैं । इसके अलावा, लंबी-तरंग क्षेत्र में दीवार फिसलन को बढ़ाने से, इंटरफेस मोड को स्थिर किया जा सकता है, लेकिन सतह मोड को अस्थिर किया जा सकता है जब $m < 1$ । हालांकि, इंटरफेस और सतह मोड पर दीवार फिसलन का प्रभाव $m > 1$ में पूरी तरह से विपरीत होता है। इसके अलावा, श्यानता अनुपात सतह मोड द्वारा उत्पन्न प्राथमिक अस्थिरता में एक दोहरी भूमिका प्रदान करता है, अर्थात, यह न्यूनतम सीमा मूल्य के करीब एक स्थिर प्रभाव प्रदर्शित करता है, लेकिन न्यूनतम सीमा मूल्य से दूर एक अस्थिर प्रभाव प्रदर्शित करता है। हालांकि, सतह मोड के बारे में उपरोक्त परिणाम पूरी तरह से विपरीत हैं यदि श्यानता अनुपात के जगह

घनत्व अनुपात, या, मोटाई अनुपात परिवर्तित होता है। इसके अलावा, इंटरफेस पृष्ठसक्रियकारक मोड को घनत्व अनुपात, श्यानता अनुपात और मोटाई अनुपात के परिमाण को बढ़ाकर स्थिर किया जा सकता है। इसके अलावा, अपरूपण मोड संख्यात्मक अनुकरण में दिखाई देते हैं जब रेनॉल्ड्स संख्या बहुत बड़ी होती है और झुकाव कोण बहुत छोटा होता है। निचली द्रव परत के साथ जुड़े अपरूपण मोड को स्थिर किया जा सकता है, लेकिन ऊपरी तरल परत के साथ जुड़े अपरूपण मोड को श्यानता अनुपात के मूल्य में वृद्धि करके अस्थिर किया जा सकता है।

Contents

Certificate	i
Acknowledgements	iii
Abstract	v
Contents	ix
List of Figures	xv
List of Tables	xxi
List of Abbreviations	xxiii
List of Symbols	xxv
1 General Introduction	1
1.1 Hydrodynamic Stability	3
1.2 Applications	4
1.3 Literature Survey	4
1.3.1 Single-layer	4
1.3.2 Two-layer	8
1.4 Research Gap and Motivation	11
1.5 Objectives	13
1.5.1 Objective 1: Linear stability of single layer surfactant laden free surface flow.	13
1.5.2 Objective 2: Linear stability of a two-layer surfactant-laden film flows.	14
1.6 Organization of the Thesis	15
2 Methodologies	17

2.1	Governing Equations and Boundary Conditions	17
2.2	Orr-Sommerfeld Equation	22
2.3	Solution Methodologies	23
2.3.1	Inertia-less analysis	24
2.3.2	Long-wave analysis	25
2.3.3	Arbitrary wave-number analysis	27
2.3.4	Inviscid analysis	29
2.3.5	Energy budget	29
3	Linear Stability of a Contaminated Fluid Flow Down a Slippery In-	
	clined Plane	33
3.1	Introduction	33
3.2	Equations of Motion	34
3.2.1	Dimensionless criterion	38
3.2.2	Perturbation equations	39
3.3	Orr-Sommerfeld Equation OSE	40
3.4	Solution of OSE at Inertia-less Approximation Without Surfactant	42
3.5	Solution of OSE at Low Reynolds Number	44
3.6	Solution of OSE for Arbitrary Reynolds Number	50
3.6.1	Solution of OSE at moderate Reynolds number	50
3.6.2	Spectrum of the Orr-Sommerfeld eigenvalue problem . . .	58
3.6.3	Convergence of spectrum	59
3.6.4	Effect of slip length on the shear mode	62
3.6.5	Effect of Marangoni number on the shear mode	64
3.6.6	Effect of Péclet number on the shear mode	65
3.6.7	Competition between surface and shear modes	67
3.7	Inviscid Stability Analysis Without Surfactant	72

3.8	Energy Budget Analysis	74
3.9	Conclusions	78
4	Linear Stability Analysis of a Surfactant-Laden Shear-Imposed Falling Film	81
4.1	Introduction	81
4.2	Mathematical Formulation	82
4.3	Orr-Sommerfeld Boundary Value Problem	85
4.4	Long-wave Stability Analysis	86
4.4.1	Zeroth order solution	86
4.4.2	First order solution	88
4.4.3	Second order solution	93
4.4.4	Third order solution	95
4.5	Energy Budget for a Shear-imposed Flow	98
4.6	Linear Stability Analysis for Moderate Reynolds Number	103
4.6.1	Surface mode	104
4.6.2	Surfactant mode	108
4.7	Linear Stability Analysis for High Reynolds Number	110
4.8	Linear Stability Analysis in Stokes Limit	114
4.9	Discussions and Conclusions	118
5	Linear Stability for Surfactant-laden Two-layer Film Flowing Down a Slippery Inclined Plane	121
5.1	Introduction	121
5.2	Mathematical Formulation	122
5.2.1	Dimensionless criterion and linearized equations	129
5.3	Orr-Sommerfeld Boundary Value Problem (OS BVP)	131
5.4	Study of Stability Analysis for Arbitrary Wave-number	137

5.4.1	Effect of density ratio on SM, IM and ISM in the presence of slip length	142
5.4.2	Effect of viscosity ratio on SM, IM and ISM in the presence of slip length	151
5.4.3	Effect of thickness ratio on SM, IM and ISM in the presence of slip length	155
5.4.4	Effect of wall slip on SM, IM and ISM	158
5.4.5	Results of shear modes for a two-layer film flows with a free surface	161
5.5	Conclusions	163
6	Conclusion	167
6.1	Introduction	167
6.2	Summary of Present Work	169
6.2.1	Linear stability of a contaminated fluid flowing down a slippery inclined plane	169
6.2.2	Linear stability analysis of a surfactant laden shear-imposed falling film	171
6.2.3	Linear stability for surfactant-laden two-layer film flowing down a slippery inclined plane	173
6.3	Future Work	176
	Appendices	177
A	Expressions for Longwave Results.	179
A.1	Longwave Results for "Contaminated Fluid Flow Down a Slippery Inclined Plane"	179
A.1.1	Zeroth order	179
A.1.2	First order	179

A.1.3	Second order	180
A.1.4	Third order	180
A.2	Longwave Results for “Surfactant-Laden Shear-Imposed Falling Film”	182
A.2.1	Zeroth order	182
A.2.2	First order	182
A.2.3	Second order	183
A.2.4	Third order	184
A.3	Longwave Results for “Surfactant-laden Two-layer Film Flowing Down a Slippery Inclined Plane”	187
A.3.1	Zeroth order	187
A.3.2	First order	188
	Bibliography	193
	List of Publications	207
	Author Bio-data	209

List of Figures

1.1	Instabilities on water flowing down a slightly slopping road in rainy season [2].	2
1.2	(a) Curtain coating [17]. (b)Thin film evaporator [18].	5
2.1	Schematic diagram of a gravity-driven free surface flow.	18
2.2	Schematic diagram of slippage phenomenon near the slippery substrate with slip length α . (redrawn from Blake [42])	20
2.3	Schematic geometry of the free surface of a fluid flow.	21
2.4	Transport of insoluble surfactant at the free surface in time δt	21
2.5	Flowchart for plotting the growth rate. k_{start} and k_{end} denote the starting and end values of wave-number k	28
2.6	Flowchart for performing the energy budget analysis	30
3.1	Schematic diagram of a surfactant-laden fluid flow down a slippery inclined plane.	34
3.2	Variation of streamwise base velocity and shear stress.	37
3.3	Variation of the critical Reynolds number Re_c for the surface mode with slip length	48
3.4	Neutral curves and growth rate plots depicting effect of slip on the surface mode	51
3.5	Neutral curves and growth rate plots depicting effect of Ma on the surface mode	52

3.6	Neutral curves and growth rate plots depicting effect of Péclet number on the surface mode	53
3.7	Variation of the spatial growth rates for different values of Péclet numbers	54
3.8	Spectrum of the eigenvalue problem (3.76) when $Re = 21000$. . .	57
3.9	Spectrum of the eigenvalue problem (3.76) for different values of the slip length when $Re = 30000$	59
3.10	Spectrum of the eigenvalue problem (3.76) for different values of the slip length when $Re = 80000$	60
3.11	Variation of the relative error E_N with the number of Chebyshev polynomials N	61
3.12	Neutral curves and growth rate plots depicting effect of slip on the shear mode	62
3.13	Neutral curves and growth rate plots depicting effect of Marangoni number on the shear mode	63
3.14	Neutral curves and growth rate plots depicting effect of Péclet number on the shear mode	66
3.15	Competition between the surface and the shear mode for different values of slip.	68
3.16	Competition between the surface and the shear mode for different values of Marangoni number.	69
4.1	The schematic diagram of a surfactant-laden shear-imposed fluid flowing down an inclined plane.	82
4.2	Variation of critical Reynolds number with Marangony number and imposed shear.	89

4.3	Variation of the critical Péclet number Pe_{cm} for the surfactant mode with the imposed shear stress for different values of Ma . . .	92
4.4	Variation of the temporal growth rate kc_i for the surfactant mode with τ and Ma	95
4.5	Variation of the temporal growth rate kc_i for the surfactant mode and Phase boundary between unstable and stable regions in (τ, Ma) plane	96
4.6	Variation of the rate of kinetic energy, rate of work done by the perturbation shear stress, rate of viscous dissipation work and work done by the imposed shear stress with k	101
4.7	Variation of the neutral curve and growth rates for the surface mode depicting effect of Marangoni number	104
4.8	Variation of the neutral curve and growth rates for the surface mode depicting effect of imposed shear (τ)	106
4.9	Comparison between numerical and analytical results.	107
4.10	Variation of the neutral curve and growth rates for the surfactant mode depicting effect of Marangoni number	108
4.11	Variation of the neutral curve and growth rates for the surfactant mode depicting effect of imposed shear	110
4.12	Variation of the neutral curve and growth rates for the surfactant mode depicting effect of imposed shear	111
4.13	Variation of the neutral curve and growth rates for the surfactant mode depicting effect of Marangoni number	112
4.14	Phase boundary between the stable "S" and unstable "U" zones in (τ, Ma) plane	114

4.15	Variation of the neutral curves for the surface and shear modes in the (Re, k) plane for different values of τ	115
4.16	Variation of the temporal growth rate kc_i for the surfactant mode with wave-number k for several values of τ and Ma	117
4.17	Variation of the neutral curve and growth rates for the surfactant mode in the inertia-less limit ($Re = 0$)	118
5.1	Schematic diagram for the surfactant-laden two-layer film flow- ing down a slippery inclined plane.	123
5.2	Variation of base flow velocities when the flow parameters vary .	128
5.3	Impact of density ratio on the surface mode.	138
5.4	Variation of the critical Reynolds number for the interface mode with density ratio.	139
5.5	Impact of density ratio on the interface mode when $mr < 1$	140
5.6	Impact of density ratio on the interface mode when $mr > 1$	141
5.7	Impact of density ratio on the interface surfactant mode (ISM) . .	143
5.8	Impact of viscosity ratio on the surface mode	144
5.9	Impact of viscosity ratio on the interface mode.	145
5.10	Impact of viscosity ratio on the interface surfactant mode (ISM) .	147
5.11	Impact of depth ratio on the surface mode (SM)	148
5.12	Impact of depth ratio on the interface mode (IM)	149
5.13	Impact of depth ratio on the interface surfactant mode (ISM) . . .	150
5.14	Impact of slip length on the surface mode (SM)	151
5.15	Impact of slip length on the interface mode	153
5.16	Impact of slip length on the interface surfactant mode (ISM) . . .	154
5.17	Scenario of stability boundaries for SM, IM and ISM and neutral curve for shear mode.	156

5.18 Impact of viscosity ratio on the shear mode.	159
5.19 Impact of slip length on the shear mode.	160

List of Tables

3.1	Comparison between the analytical and numerical values of the critical Reynolds number for the surface mode for different values of the slip length when $\theta = \pi/4$, $Ca = 2$, $Pe = 2$ and $Ma = 1$.	50
3.2	Comparison between the analytical and numerical values of the critical Reynolds number for the surface mode for different values of the Marangoni number when $\theta = \pi/4$, $Ca = 2$, $Pe = 2$ and $\beta = 0$.	50
3.3	The analytical values of various terms in energy budget equation (3.89) when the wave-number varies. The other flow parameters $\theta = 60^\circ$, $Re = 20$, $Ca = 1$, and $\beta = 0.025$ are fixed.	77
3.4	The analytical values of various terms in energy budget equation (3.89) when the slip length varies. The other flow parameters $\theta = 60^\circ$, $Re = 20$, $Ca = 1$, and $k = 0.05$ are fixed.	78
4.1	The magnitude of the ratio ($ She/Dis $) when the wave-number k varies.	98
4.2	Numerical values of different terms in the energy budget equation (4.59) when the wave-number k varies.	99
5.1	Physical properties of immiscible fluid layers.	127
5.2	Expression of the critical Reynolds number Re_{1cSM} for the surface mode in different limits.	132

5.3	Numerical and analytical values of the complex wave speed c for the interface mode	134
5.4	Numerical and analytical values of the complex wave speed c for the surface mode	135
5.5	Numerical and analytical values of the complex wave speed c for the interface mode	136
5.6	Numerical and analytical values of the complex wave speed c for the surface mode	138
6.1	Characteristics of different modes found in the study of linear stability of a contaminated fluid flow down a slippery inclined plane.	171
6.2	Characteristics of different modes found in the study of linear stability analysis of a surfactant-laden shear-imposed falling film.	173
6.3	Characteristics of different modes found in the study of linear stability for surfactant-laden two-layer film flowing down a slippery inclined plane.	175

List of Abbreviations

OSE	Orr Sommerfeld Equations
OS BVP	Orr Sommerfeld Boundary Value Problem
SM	Surface Mode
IM	Interface Mode
SSM	Surface Surfactant Mode
ISM	Interface Surfactant Mode

List of Symbols

c	Complex wave speed	m s^{-1}
d	Height of undisturbed film	m
Ca	Capillary Number	–
D_s	Diffusivity of surfactant	$\text{m}^2 \text{s}^{-1}$
E	Surface elasticity	–
E_k	Kinetic energy of disturbances	J
g	Gravitational acceleration	m s^{-2}
h	Height of disturbed film	m
k	Wave-number	m^{-1}
Ka	Kapitza number	–
m	Viscosity ratio	–
Ma	Marangony Number	–
p	Pressure field	N m^{-2}
Pe	Péclet Number	–
r	Density ratio	–
Re	Reynolds Number	–
Re_c	Critical Reynolds Number	–
Pe_c	Critical Péclet Number	–
t	Time	s
T_n	n^{th} degree Chebyshev polynomial of first kind	–
U	Base flow velocity	m s^{-1}
V	Velocity vector	m s^{-1}
Q	2D Base flow rate	$\text{m}^2 \text{s}^{-1}$
We	Weber Number	–
α	Slip length	m
β	Non-dimensional slip length	–
δ	Thickness ratio	–
γ	Non-dimensional amplitude of surfactant concentration	–
η	Non-dimensional amplitude of free surface deformation	–
λ	Wavelength of disturbances	m
μ	Dynamic viscosity	Pa s
ρ	Density of fluid	kg m^{-3}
σ	Surface tension	N m^{-1}
ψ	Non dimensional stream-function	–
ϕ	Non dimensional amplitude of stream-function	–
ω	Angular frequency	rad/s
∂_x	Partial derivatives with respect to x	–

Γ	Surfactant concentration	–
τ	Stress tensor	Nm^{-2}
τ_s	Imposed shear	Nm^{-2}
θ	Angle of inclination	rad