

IMPROVED MODEL ORDER REDUCTION
STRATEGIES FOR A CLASS OF NONLINEAR
CIRCUITS

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IMPROVED MODEL ORDER REDUCTION STRATEGIES FOR A CLASS OF NONLINEAR CIRCUITS

by

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CERTIFICATE

This is to certify that the thesis entitled “**Improved Model Order Reduction Strategies for a Class of Nonlinear Circuits**”, submitted by **Shifali Kalra** to the Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy** in Electrical Engineering, is a record of the bonafide research work carried out by her under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the award of the degree.

The results contained in this thesis have not been submitted either in part or in full to any other University or Institute for the award of any degree or diploma to the best of my knowledge.



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ABSTRACT

In many application areas, experimental analysis are been increasingly replaced by numerical simulations in order to save design and development costs. However, modelling physical systems accurately often result into large order dynamical systems that are computationally heavy to simulate. Thus, approximation becomes important in order to have cost-effective simulation. There are different model order reduction (MOR) techniques that can be used to reduce the order of large dynamical systems. MOR results in a system that is dimensionally reduced and has input-output characteristics almost similar to the large order original system. MOR techniques have been widely used by many researchers across disciplines to successfully reduce large order systems. There are different MOR techniques depending upon the type of system. The two broad categories of MOR techniques for linear systems are based on singular value decomposition (SVD) and on Krylov subspaces. Similarly, there are quite a few MOR techniques that are used for nonlinear systems like proper orthogonal decomposition (POD), trajectory piecewise linear approximation (TPWL) etc. The focus of this thesis is on TPWL and its variants.

The TPWL method involves first selection of few linearisation points (LPs) on the system trajectory. The system is then locally linearised at those LPs and reduced using linear MOR technique. The weighted summation of the reduced models is done to obtain the final TPWL approximation. Although TPWL and its variants are very popular among researchers and are widely used as nonlinear order reducing tools, they possess some issues especially with the LP selection that needs attention. Selection of LPs is a crucial step as it decides the effectiveness of the overall approximation. The quality of the approximation does not always depend upon the number of the LPs only but also on the correct placement of LPs on the system trajectory. The conventional approach of deciding the LPs included the uniform division of the system trajectory at a constant preselected distance, often resulting into inadequate sampling. Moreover, the user

defined thresholds make the approximation heuristic in nature.

The major concern of this thesis focuses on identifying issues related to the conventional LP selection in TPWL and addressing them by proposing new criteria that are capable of judiciously placing LPs on the system trajectory thereby generating better approximations. Later in the thesis, criteria is proposed that not only removes the ambiguity associated with the TPWL method, but also generates better approximations with far less number of sub-models. The proposed schemes have been tested and simulated on few nonlinear circuits including transmission line circuit, RC ladder network and inverter chain circuit.

Keywords: Large order systems, Nonlinear circuits, Model order reduction, Trajectory piecewise linear approximation, Linearisation points selection.

सार

कई एप्लिकेशन क्षेत्रों में, डिजाइन और विकास लागतों को बचाने के लिए संख्यात्मक विश्लेषणों की तुलना में प्रयोगात्मक विश्लेषण तेजी से बदल रहे हैं। हालाँकि, मॉडलिंग भौतिक प्रणालियां अक्सर बड़े क्रम के डायनेमिक सिस्टम में परिणत होती हैं जो अनुकरण करने के लिए कम्प्यूटेशनल रूप से भारी होती हैं। इस प्रकार, लागत प्रभावी अनुकरण करने के लिए सन्निकटन महत्वपूर्ण हो जाता है। विभिन्न मॉडल ऑर्डर रिडक्शन (एमओआर) तकनीकें हैं जिनका उपयोग बड़े डायनेमिक सिस्टम के ऑर्डर को कम करने के लिए किया जा सकता है। एमओआर एक ऐसी प्रणाली का परिणाम देता है जो मंद रूप से कम होती है और इसमें इनपुट-आउटपुट विशेषताएँ होती हैं जो बड़े ऑर्डर की मूल प्रणाली के समान होती हैं। एमओआर तकनीकों का व्यापक रूप से कई शोधकर्ताओं द्वारा बड़े ऑर्डर सिस्टम को सफलतापूर्वक कम करने के लिए उपयोग किया जाता है। प्रणाली के प्रकार के आधार पर अलग-अलग एमओआर तकनीकें हैं। रैखिक प्रणालियों के लिए एमओआर तकनीकों की दो व्यापक श्रेणियां एकल मूल्य अपघटन (एसवीडी) और क्रिलोव उप-स्थानों पर आधारित हैं। इसी तरह, काफी कुछ एमओआर तकनीकें हैं जिनका उपयोग गैर-रैखिक प्रणालियों के लिए किया जाता है जैसे कि उचित ऑर्थोगोनल अपघटन (पीओडी), प्रक्षेपवक्र टुकड़ा-रहित रैखिक सन्निकटन (टीपीडब्ल्यूएल) आदि। इस थीसिस का फोकस टीपीडब्ल्यूएल और इसके वेरिएंट पर है।

टीपीडब्ल्यूएल पद्धति में सिस्टम प्रक्षेपवक्र पर कुछ रैखिककरण बिंदुओं (एलपी) का पहला चयन शामिल है। तब सिस्टम स्थानीय रूप से उन एलपी में रैखिककृत होता है और रैखिक एमओआर तकनीक का उपयोग करके कम किया जाता है। कम किए गए मॉडल का भारित योग अंतिम टीपीडब्ल्यूएल सन्निकटन प्राप्त करने के लिए किया जाता है। यद्यपि टीपीडब्ल्यूएल और इसके वेरिएंट शोधकर्ताओं के बीच बहुत लोकप्रिय हैं और व्यापक रूप से गैर-रेखीय आदेश को कम करने वाले उपकरणों के रूप में उपयोग किया जाता है, वे विशेष रूप से एलपी चयन के साथ कुछ मुद्दों के अधिकारी हैं जिन्हें ध्यान देने की आवश्यकता है। एलपी का चयन एक महत्वपूर्ण कदम है क्योंकि यह समग्र सन्निकटन की प्रभावशीलता को तय करता है। सन्निकटन की गुणवत्ता हमेशा एलपी की संख्या पर ही निर्भर नहीं करती है, बल्कि सिस्टम विश्लेषण पर एलपी के सही स्थान पर भी निर्भर करती है। एलपी तय करने के पारंपरिक दृष्टिकोण में एक निरंतर अनियंत्रित दूरी पर प्रणाली प्रक्षेपवक्र का एकसमान विभाजन शामिल था, जिसके परिणामस्वरूप अक्सर

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इस थीसिस की प्रमुख चिंता टीपीडब्लूएल में पारंपरिक एलपी चयन से संबंधित मुद्दों की पहचान करने और नए मानदंड प्रस्तावित करके उन्हें संबोधित करना है जो सिस्टम प्रक्षेपवक्र पर एलपी को निर्णायक रूप से रखने में सक्षम हैं जिससे बेहतर सन्निकटन उत्पन्न होते हैं। बाद में थीसिस में, मानदंड प्रस्तावित किया जाता है कि न केवल टीपीडब्लूएल पद्धति से जुड़ी अस्पष्टता को दूर करता है, बल्कि उप-मॉडल की कम संख्या के साथ बेहतर सन्निकटन भी उत्पन्न करता है। प्रस्तावित योजनाओं को ट्रांसमिशन लाइन सर्किट, आर सी सीढ़ी नेटवर्क और इन्वर्टर चेन सर्किट सहित कुछ नॉनलाइनियर सर्किट पर परीक्षण और अनुकरण किया गया है।

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Nomenclature

Acronyms	Description
AFAS	Adaptive Fast Approximate Simulation
BT	Balanced Truncation
DEIM	Discrete Empirical Interpolation
DMD	Dynamic Mode Decomposition
FAS	Fast Approximate Simulation
FEM	Finite Element Method
GMEC	Global Maximum Error Controller
HNA	Hankel Norm Approximation
LP	Linearisation Point
LTI	Linear Time Invariant
MOR	Model Order Reduction
NTPWL	Nonlinearity aware Trajectory Piecewise Linear
ODE	Ordinary Differential Equation
PDE	Partial Differential Equation
PM	Projection Matrices
POD	Proper Orthogonal Decomposition
ROM	Reduced Order Model
SAAS	Subspace Angle based Adaptive Sampling

Continued from previous page

Acronyms	Description
SVD	Singular Value Decomposition
TBR	Truncated Balanced Realisation
TPWL	Trajectory Piecewise Linear
UDET	Uniform Division of Exact Trajectory

List of Symbols	Description
x	State vector of full order dynamical system
A	State matrix
B	Input matrix
C	Output matrix
n	Full order of the dynamical system
r	Order of ROM
\mathbb{R}^n	The n -dimensional real vector space
\in	Belongs to
u	Input to the dynamical system
y	Output of the dynamical system
s	Number of inputs to dynamical system
o	Number of outputs to dynamical system
V	Projection subspace
z	State vector of reduced model
A_r	State matrix of reduced model
B_r	Input matrix of reduced model

List of Symbols	Description
C_r	Output matrix of reduced model
P	Controllability gramian
Q	Observability gramian
$K_q(A, b)$	Krylov subspace
q	Positive integer
b	Starting vector
H	Hessenberg
$f(x)$	Nonlinear vector field
ϵ	Radius of trust region
m	Number of LPs or sub-models
A_i	Jacobian of $f(x)$
w_i	Weights assigned to sub-models
δ	Euclidean distance
β, γ, η	Positive constants
p_i	Participation measure
$\theta_{i,i+1}$	Angle between subspaces
θ_{\max}	Maximum tolerance of subspace angle
$\lceil \cdot \rceil$	Ceiling function
e_{output}	Error in output
e_{state}	Error in states
y_{FL}	Output of full order nonlinear system
y_{red}	Output of ROM
$H(t)$	Heaviside unit step input
R	Resistance

List of Symbols**Description** C_p

Capacitance

 α

Maximum error limit

 I

Identity matrix