

**SOME CONTRIBUTIONS TO THE HIGH RESOLUTION
ESTIMATION OF DIRECTIONS-OF-ARRIVAL
OF PLANE WAVES**

by
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*Thesis submitted
in fulfilment of the requirements
for the award of the degree of
DOCTOR OF PHILOSOPHY*



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CERTIFICATE

This is to certify that the thesis entitled "**Some Contributions to the High Resolution Estimation of Directions-of-Arrival of Plane Waves**", being submitted by Bindu Chandna to the Department of Electrical Engineering, Indian Institute of Technology, New Delhi, for the award of degree of Doctor of Philosophy, is a bona-fide research work carried out by her under our supervision and guidance. The results contained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.



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Abstract

High Resolution Array processing is important in many signal processing systems such as radar, sonar, electronic surveillance, tomography etc. The objective of array processing is to extract some useful characteristics of the received signal field, e.g., the Directions-of-Arrival (DOA) of the signals, the signal strengths, the number of signals that are correlated with each other and the total number of signals. A number of algorithms based on the eigen-structure of the array covariance matrix have been proposed. Of these, MUSIC (Multiple Signal Classification) and ESPRIT (Estimation of Signal Parameters via Rotational Invariant Technique) are the two most popular techniques.

These techniques work well in the presence of uncorrelated or partially correlated sources. In many practical situations, however, the various sources present in the field are fully correlated with each other and these techniques then, encounter difficulties. To overcome this difficulty, pre-processing techniques like spatial smoothing have been proposed for some of the techniques like MUSIC. The optimum use of spatial smoothing, however, requires a complete knowledge regarding the number of sources that are coherently related to each other and the total number of sources present in the field. Furthermore, these pre-processing techniques, need to be developed for other currently interesting algorithms, like ESPRIT, whose performance is seen to degrade significantly in the presence of partially correlated sources and breaks down completely in the presence of coherent multipath signals.

In the present work, an attempt has been made to explore some of the current topical aspects of the DOA estimation problem. The issues taken up for study here include: (a) computationally efficient alternatives to the complete eigen-decomposition which offer a graceful trade-off of performance with computational burden; (b) alternative, possibly more efficient methods of determining the number of coherent sources, required for pre-processing techniques, than those currently available and (c) devising pre-processing techniques for use with algorithms like ESPRIT in the presence of coherent sources. In addition, we also present a general framework for the DOA estimation via "state space" techniques, which are shown to offer several alternatives that share some interesting properties with ESPRIT, besides being easier

amenable to pre-processing required for coherent sources.

Algorithms like MUSIC require the computation of an eigen-decomposition of the array covariance matrix, which is a computationally expensive operation. We here suggest a procedure that alleviates this difficulty by the use of Householder transformations of the appropriately permuted array covariance matrix. The computational complexity of this algorithm is significantly smaller than that of a complete eigen-decomposition. Due to the multipath propagation or due to the scenarios involving smart jammers, the various signals impinging on the array are mutually coherent. Under these situations, pre-processing techniques like spatial smoothing are used to decorrelate the coherent sources. The optimum use of spatial smoothing, however, requires prior knowledge regarding the source coherency structure (SCS) (i.e. the number of groups of coherent sources and the number of sources per group). We propose here an efficient method for determination of the SCS and the total number of sources present in the field, which improves the performance of the existing, so called Smoothed Rank Profile (SRP) procedure.

In recent years, ESPRIT has evolved as a powerful alternative to MUSIC because of the advantages it offers both in terms of the computational efficiency as well as performance. In our work here, we suggest methods of invoking pre-processing techniques like "spatial smoothing" in the ESPRIT framework in order to make ESPRIT work in the presence of coherent sources. It is shown that this can be done via the use of an appropriate form of smoothing of both the auto and the cross covariance matrices. To improve the coherent signal resolving capability further, we also suggest the use of "combined forward-backward averaging and spatial smoothing" of the auto and cross covariance matrices in the ESPRIT framework.

Finally, we propose and formulate a "spatial" state variable formalism for the problem of DOA estimation. These are motivated by the fact that information regarding the source directions is contained in the eigenvalues of the so-called state transition matrix of the such a "spatial" state variable description. The problem of estimating the DOAs can therefore be reduced to that of identification of this matrix, for which a host of techniques become available from the system identification literature. These techniques are also shown to incorporate a different kind of spatial smoothing, making them also easily applicable for the case of coherent sources.

Interesting relationships are shown to exist between these state variable techniques and ESPRIT, both in the situations of uncorrelated and coherent source environments. Extensive simulation results are presented to develop insight for and compare the performance of the various techniques of interest in this dissertation.

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