

**SOME PROBLEMS ON THE DECAY OF
HOMOGENEOUS TURBULENCE**

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C E R T I F I C A T E

This is to certify that the thesis entitled 'Some Problems on the Decay of Homogenous Turbulence' which is being submitted by Mr. Joga Singh Rawat for the award of Degree of Doctor of Philosophy (Mathematics) to the Indian Institute of Technology, Delhi, is a record of bonafide research work. He has worked for the last five years under my guidance and supervision.

The thesis has reached the standard of fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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A C K N O W L E D G E M E N T S

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S Y N O P S I S

This thesis incorporating some aspects of the statistical features of the decay of homogeneous turbulence comprises of six chapters. The first chapter is devoted to the introduction which includes, in brief, the concept of turbulence, the necessity of statistical approach for discussing the turbulent phenomena and the justification for its use, the present-day scope of turbulence, the gradual development of some related ideas and outstanding contributions of some of the pioneers in the field.

In the second chapter, a two-point, two-time approach is attempted for the final period of decay of turbulence in an external, homogeneous magnetic field. Two-point, two-time correlation and spectral equations are obtained by considering the equations of fluid and electrodynamics for two points in a turbulent fluid at two different times. Solutions, corresponding to any given initial distribution, are obtained by assuming that the turbulence is weak enough for second-order truncation approximations to be applicable. The analysis shows that pronounced axisymmetric properties are developed in this case, turbulence elements with small extensions in the direction of the field and with large time separations being damped relatively rapidly under

normal physical conditions. In its final period, the decay is governed by both periodic and non-periodic motions.

The third chapter is devoted to examining the final period of decay of magneto-fluid dynamic turbulence in an incompressible, electrically conducting fluid subjected to an external, homogeneous magnetic field and the Coriolis force resulting from a rotation of stationary angular velocity inclined to the field. A system of two-point correlation and spectral equations for the problem is constructed from the equations of fluid and electrodynamics. Explicit solutions for various spectral quantities are obtained by assuming that the turbulent field is homogeneous and weak enough for triple correlations to be negligible. It is shown that the effect of Coriolis force is to counteract the influence of the magnetic field on the decay. The axisymmetry, developed in the absence of a Coriolis force, is destroyed and the damping effects and periodicity are modified.

The fourth chapter presents an analysis of the decay of homogeneous hydromagnetic turbulence for times before the final period of decay. The work reported here is essentially an extension of the previous analyses, successfully attempted, for ordinary velocity and scalar fields

to the case of magneto-fluid dynamic turbulence but this effort has met a limited success. The method consists of finding the solutions of 2 - ^{and} k - point Fourier-transformed equations after neglecting the fourth-order correlations in comparison with the second- and third-order correlations. The solutions could be obtained only for the case when kinematic viscosity equals the magnetic diffusivity. Thus, corresponding to this case, results are obtained for kinetic energy spectrum function, magnetic energy spectrum function and the transfer function which are applicable for times considerably before, as well as during, the final period.

In the fifth chapter, the effects of buoyancy, produced by a uniform vertical concentration gradient and body force, on a homogeneous turbulent field accompanied by a first-order chemical reaction, are analysed by considering a simplified model. A system of two-point correlation equations, which contains mean concentration gradient and body force terms, is constructed from the Navier-Stokes, convective diffusion and continuity equations. By well-known methods, these equations are converted into equations for the spectrum functions in wave-number space and solutions for different spectral tensors are obtained by neglecting the contributions of the triple correlation terms. For carrying out the

numerical calculations, it is assumed that the turbulence is initially isotropic and the concentration fluctuations initially zero. It turns out that the turbulence decays with time, although buoyancy forces do alter the rate of decay. The buoyancy forces can either extract energy from the turbulent field or feed energy into it, depending upon the direction of the body force and the concentration gradient. Spectra are displayed graphically for several values of the reaction rate parameter for stabilizing as well as destabilizing buoyancy forces.

The sixth and the concluding chapter, deals with the problem of steady-state locally homogeneous but longitudinally varying turbulence in an incompressible flow. The statistical behaviour of longitudinal mass transfer is discussed for turbulent concentration fields undergoing first-order chemical reaction. The restrictions imposed are that the velocity field remains unaffected by the chemical reaction and local mass transfer, and that the reaction rate "constants" and the diffusivity remain constant in space-time. Under these restrictions, a first-order analysis of the turbulent field which retains the effects of viscosity, molecular diffusivity, reaction rate and mean strain, is carried out by employing the usual technique

of Deissler. The different quantities characterizing the longitudinal mass transfer are deduced for two cases: firstly, for cases which include the effects of strain and, secondly, for cases of pure viscous decay. The calculations are made for both, accelerating as well as decelerating, flows by using the already familiar initial conditions.

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CERTIFICATE

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