

# **DYNAMIC STABILITY ANALYSIS OF COMPLIANT OFFSHORE STRUCTURES**

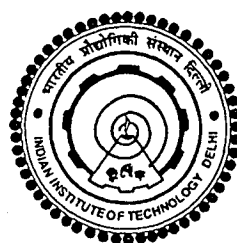
By

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Submitted

in fulfilment of the requirements of the degree of  
**DOCTOR OF PHILOSOPHY**

to the



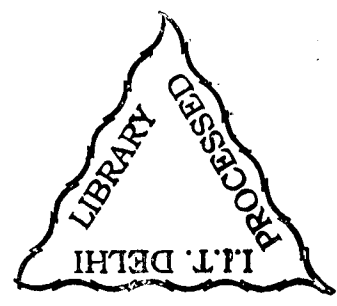
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## CERTIFICATE

This is to certify that the thesis entitled, “**Dynamic Stability Analysis of Compliant Offshore Structures**”, being submitted by **Mr. Atul Krishna Banik**, to the Indian Institute of Technology, Delhi, for the award of ‘**DOCTOR OF PHILOSOPHY**’ in Civil Engineering is a record of the bonafide research work carried out by him under my supervision and guidance. He has fulfilled the requirements for submission of this thesis, which to the best of my knowledge has reached the requisite standard.

The material contained in the thesis has not been submitted in part or full to any other University or Institute for the award of any degree or diploma.



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April, 2004

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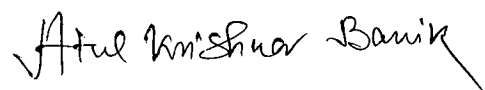
I express my deep gratitude to my parents and elders for their blessings and encouragement that helped me throughout my research work and finally, in achieving the target.

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(Atul Krishna Banik)

## ABSTRACT

Dynamic stability analysis of nonlinear systems has received considerable interest among the researchers in the last decade or so. Various methods of dynamic stability analysis of nonlinear systems in closed form and using analytical, semi analytical and numerical techniques have been developed. Application of these techniques cover a wide range of application problems including standard problem of Van-Der-Pol Oscillator, Duffing Oscillator, Double pendulum etc. One of the potential areas of application of dynamic stability analysis in civil engineering is compliant offshore structures, which are inherently nonlinear. Therefore, there have been several studies on the dynamic stability of offshore facilities especially compliant offshore structures. In spite of these studies, there is a need for more studies in this area especially to use recently developed techniques for finding out their capabilities to explore different kinds of instabilities inherent in such problems. Further, scanty literature is available on the stochastic stability of flexible offshore structures. Since, the oceanic waves are better modeled as a stochastic process, there is a need to investigate the stochastic stability of flexible offshore structures. With this background in view, the present study is undertaken. The objectives of the study are (i) to develop a comprehensive numerical scheme using incremental harmonic balance with path following continuation and Floquet theory to find the response behavior and stability of non linear systems associated with compliant offshore structures; (ii) to develop a procedure for stochastic response and stability analysis of the above type of nonlinear offshore systems; and (iii) to apply the procedure developed for finding the response and stability of a) two point and four point mooring systems; b) articulated leg platform; and c) vortex induced oscillation of TLP tethers.

In the first part of the study, the general formulation of incremental harmonic balance method with continuation technique (IHBC) is described for the nonlinear response analysis of nonlinear SDOF system and describes how nT solutions are obtained using IHBC. Also, it describes how the local stability of the solution is investigated using Floquet's theory. A computational scheme is outlined which integrates the above techniques for finding automatically different solution branches and testing the stability of each solution, starting with an initial conditions. Formulations of the two and four point mooring problems are then presented and derivations of equations are brought to the form where the IHBC techniques can be directly employed. Next, the nonlinear equation of motion of articulated leg platform with varying diameter of the articulated leg is obtained for harmonic wave. Relative velocity squared drag force appearing on the right hand side of the equation of motion is represented in two different forms such that velocity depended hydrodynamic damping terms (linear or nonlinear) are brought to the left hand side of the equation of motion. A procedure for treating the nonlinear hydrodynamic damping term using distribution theory is presented so that the two forms of equation are amenable to the application of IHBC. Finally, the vortex induced oscillation of TLP tether is investigated in the vicinity of lock-in condition. The problem is formulated as a SDOF problem by assuming that the first mode response of the tether dominates the motion and the vortex shedding is caused purely due to current, which may vary across the depth of the sea. Nonlinear equation of motion has only nonlinear hydrodynamic damping term; restoring force remaining purely linear. The equation of motion is reduced to the form in which IHBC technique can easily be employed.

An extensive parametric study is conducted to study the response behaviour and stability characteristics of the three offshore problems. Some of the important

conclusions of the study include i) IHBC is capable of tracing all types of period one, 2T, 3T solutions, while NI is able to provide only stable solutions; (ii) IHBC is also superior in terms of finding all bifurcation behaviour present in the system viz. fold bifurcation, symmetry breaking bifurcation and nT solutions; (iii) inclusion of hydrodynamic drag gives rise to both stable and a number of unstable solutions with different types of branching, which are otherwise not observed without hydrodynamic drag for ALP; (iv) further, stable solutions with large amplitudes are exhibited in certain frequency range when nonlinear drag is considered; (v) the motion of the TLP tether becomes unstable for small damping and lift coefficients, while for moderate damping and lift coefficients, multiple solutions appear; and (vi) the response jumps from one branch to the other at the end of the multiple solution region.

For stochastic response and stability analysis of nonlinear systems, F-P-K equation and Van-Der-Pol transformation are used for nonwhite excitation. A analytical expression for the probability density function of the amplitude of response in the transformed domain as well as the joint probability density of the actual displacement and velocity are derived. The response statistics are obtained by taking different moments of the probability density function. The stability analysis is performed by setting the excitation term to zero but retaining the parametric excitation term, since the stability of the trivial solution is generally sought in the theory of stochastic stability. The stability analysis requires the probability density function of the amplitude of response in the transformed domain (using Van-Der-Pol transformation) to be examined at the two boundaries i.e. at zero and infinity. Depending upon different categorization of the boundary condition, the system stability is identified.

The stochastic response and stability of the two-point mooring line problem is first considered. Since no parametric excitation exists and only linear damping is present

in the system, only a local stability analysis is sufficient for the system using a perturbation technique and the Infante's method. The analysis requires the mean square response of the system, which may be obtained in various ways. In the present case, the method of Van-Der-Pol transformation and FPK equation are used to obtain the probability density function of response under external excitation force. For the stability analysis of ALP, the equation of motion of the ALP, treated as a SDOF oscillator, is rewritten in a form so that Van-Der-Pol transformation and the use of F-P-K are possible for obtaining the probability density of the response quantities. For performing the stability analysis, pure external excitation arising due to water particle kinematics are removed but the parametric excitation terms due to relative velocity squared drag force (with signum function) are retained in the equation of motion. Finally, the stochastic response and stability analysis of TLP tether under vortex shedding is performed. Stochastic stability of TLP tether under vortex-induced oscillation is obtained using the same technique as before.

Results of the numerical examples show that (i) the stationary probability density functions obtained by the proposed method for ALP and mooring system compare well with those obtained by simulation procedure; (ii) for ALP and TLP tether, the diffusion and drift exponents and the character values at the two boundaries are such that the system is asymptotically stable at the two boundaries; and (iii) for the mooring system, the local stability condition is represented by an inequality condition expressed in term of the damping ratio, initial frequency and mean square value of the response.

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