# STABILITY ANALYSIS OF TIME-DELAY SYSTEMS WITH BOUNDED NONLINEARITY

*by*

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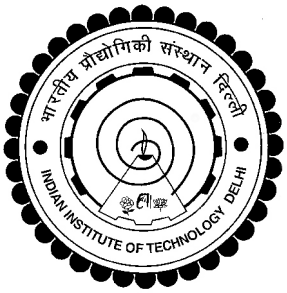
*to the*

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# CERTIFICATE

This is to certify that the thesis entitled **Stability of Time-Delay Systems with Bounded-Nonlinearity** submitted by **Pankaj Mukhija** to the Indian Institute of Technology Delhi, for the award of the Degree of Doctor of Philosophy, is a record of the original bona fide research work carried out by him under our supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted either in part or in full to any other University or Institute for the award of any degree or diploma.

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# Abstract

Most of the physical systems have time-delay and are inherently nonlinear in nature. Thus, it becomes essential to incorporate the effect of delay and nonlinearity in the stability analysis. However, the presence of delay and nonlinearity in a system complicates the stability and performance analysis. Nonlinearity in a wide variety of physical systems can be modeled as sector-bounded. Further, time-delay can be constant or time-varying. The overall aim of this work is to develop stability conditions for time-delay systems with sector-bounded nonlinearity and explore the application potential of these results to the synchronization problem. The stability conditions are developed by using the Lyapunov-Krasovskii (L-K) functional approach and are expressed in terms of Linear Matrix Inequalities (LMI).

The two broad classes of sector-bounded nonlinear systems considered in this thesis are: Lurie system and Discrete-time system with overflow and quantization nonlinearities. To begin with, an introduction to time-delay system and sector-bounded nonlinear systems, a relevant literature survey identifying research problems and main contributions of this thesis have been presented. In the first problem considered in this thesis, the probability information on time varying delay has been used to derive stability criteria for Lurie system with mixed delays. Free-weighting matrices have been used for stability analysis of system with zero lower bound. For systems with interval time-varying delay, an approach utilizing Wirtinger inequality and convex combination analysis has been used. Numerical examples have been considered to show the efficacy of proposed results. A comparison of special cases of the proposed results vis-á-vis the existing ones through numerical examples show the superiority of proposed results in determining the upper delay bound for stability.

To derive a larger upper delay bound ensuring stability is a frontier area of research in time-delay systems. With this objective, the next problem considered is to develop delay-dependent stability conditions for Lurie system and time-delay system with nonlinear perturbations by employing less conservative bounding in-

equalities. A tighter bounding technique is proposed for quadratic integral terms arising in the derivative of L-K functional. This bounding technique along with the Wirtinger inequality are used to derive stability criteria. The developed results are further extended to systems with norm-bounded parametric uncertainties. In the next problem, the stability criteria for Lurie singular time-delay system are developed by using delay-decomposition approach. Jensen inequality and Wirtinger inequality have been used to derive these criteria in terms of LMIs. It has been shown through numerical examples that the Wirtinger inequality based result yields a larger upper delay bound as compared to the Jensen inequality based result.

To explore the application potential of the results proposed in this thesis, the synchronization analysis problem of two types of complex dynamical networks (CDNs), Lurie CDN and Takagi-Sugeno (T-S) fuzzy CDN, with time-delay is considered. The synchronization stability of these two types of CDNs with time-delay is addressed by adopting the stability analysis techniques presented in the earlier addressed problems in this thesis. Further, as borne by numerical examples, these synchronization criteria are found to yield a larger upper delay bound as compared to the existing ones.

Lastly, the problem of stability analysis of discrete time system with delay and subjected to combinations of overflow and quantization nonlinearities has been investigated. Two computationally efficient stability criteria as compared to the existing results have been developed. These results have been further extended to 2-D systems. The less computational burden of proposed results has been validated through numerical examples.

The thesis concludes with a summary of the proposed results and the directions for future work.

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