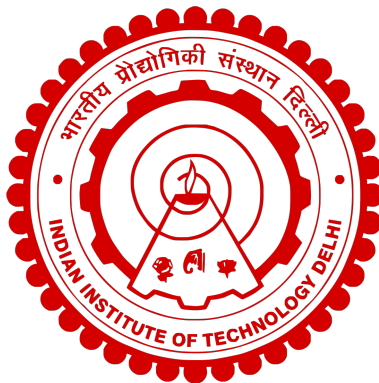


POSITION CONTROL OF FLEXIBLE LINK MANIPULATORS WITH SLIDING MODE CONTROL

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DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY DELHI
JULY 2025

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by

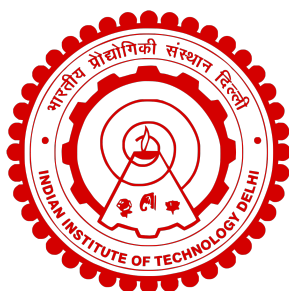
ATUL SHARMA

Submitted

in fulfillment of the requirements of the degree of Doctor of Philosophy

to the

Department of Electrical Engineering



INDIAN INSTITUTE OF TECHNOLOGY DELHI

JULY 2025

Dedicated to
My Parents, Family
and Friends

CERTIFICATE

This is to certify that the thesis entitled **Position Control of Flexible Link Manipulators with Sliding Mode Control** submitted by **Atul Sharma** to the Indian Institute of Technology Delhi, for the award of the Degree of **Doctor of Philosophy**, is a record of the bona fide research work carried out by him under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted either in part or in full to any other University or Institute for the award of any degree or diploma to the best of my knowledge.

Prof. S. Janardhanan
Department of Electrical Engineering,
Indian Institute of Technology Delhi.
(Supervisor)

ACKNOWLEDGEMENTS

First of all, I would like to express my heartfelt gratitude to my Ph.D. supervisor, Prof. S. Jannardhanan, for his insightful advice, continuous support, and steadfast encouragement throughout my Ph.D. His invaluable suggestions and constructive feedback have significantly shaped this thesis and deepened my understanding of the subject. It has been an honour to learn from his experience and work under his supervision. I also want to thank my student research committee members, Prof. Indra Narayan Kar, Dr. Deepak U. Patil, and Dr. Anima Nagar, for their constructive feedback and important suggestions, which helped me improve my knowledge of the subject and improve the quality of my work. I sincerely appreciate the efforts and time they invested in evaluating my research.

I thank Prof. Shubhendu Bhasin, Dr. Subashish Dutta, Dr. Sumit Chattopadhyay, and Dr. Priyank Srivastava for the courses delivered during my PhD coursework. I would also like to thank the late Prof. Mashuq-Un-Nabi for his invaluable guidance during the initial days of my PhD. I also want to thank Prof. Amit Kumar Jain for his constant guidance and support throughout my journey at IIT Delhi.

I will always be grateful to the Electrical Engineering Department staff for their support and assistance. I would also like to thank all my seniors, with special mention to Dr. Syed Muhammad Amrr and Apurba Das. I also thank my lab colleagues Puneet, Chaithra, Rudra, Sidhanta, Rudra Prakash, and Suman for creating an ambience for the research. Special mention to Neetish for the last-minute adjustment in the thesis.

On a personal note, I would like to remember my late parents. Without their blessings, this thesis could never have been possible. Last but not least, I would like to thank my family and in-laws, especially my elder sisters Priti and Dipti, my nieces Harshita and Tanishka, and my nephew Achal heartily, for their unconditional support and encouragement. I am deeply thankful to my support system, my better half, Supriya, for her constant support, understanding, and sacrifice throughout this journey.

Finally, I want to thank everyone who has helped me with my research and personal development, both named and unnamed. Without their encouragement and support, this thesis would not have been possible.

Atul Sharma

ABSTRACT

Flexible link manipulators represent a class of robotic manipulators known for their lightweight design. The lightweight property of the flexible link manipulators makes them suitable for various applications from land to space. However, along with the lightweight property, there is an inherent link flexibility, which introduces critical challenges in the control design, such as coupled nonlinear dynamics and vibrations at the tip, which complicates the precise tip position control. The control of multi-link flexible manipulators becomes further complicated and computationally demanding due to the interconnected link couplings compared to single link flexible manipulators.

This thesis focuses on designing position control algorithms for single link and multi-link flexible manipulators. The study begins with mathematical modelling of these systems using the Euler-Lagrange equations of motion and assumed modes method, which faithfully captures the distributed nature of link flexibility and provides a strong basis for control design. A functional observer-based sliding mode control design is proposed to effectively control single link flexible manipulators, ensuring robustness to unmodelled dynamics and disturbance. This method achieves accurate tip positioning and vibration mitigation while accommodating system uncertainties. Furthermore, advanced sliding mode control algorithms such as integral sliding mode and fast terminal sliding mode approach complemented with the observers are also proposed to obtain more substantial results in terms of stability and convergence. The control framework is extended to a multi-link flexible manipulator to address the added complexity of flexible links coupling dynamics. A novel robust fixed time integral sliding mode control technique for multi-link flexible manipulator is proposed, ensuring dynamic stability and accurate tip control over interconnected links.

Extensive numerical simulations validate the robustness and efficacy of the proposed control algorithms in tracking and regulation problems, as presented in this thesis. The results also highlight the control design extension from single link to multi-link flexible manipulator. This research advances our understanding of using robust control algorithms for flexible robotic systems, hence assisting in the design of reliable solutions for complicated, dynamic settings. Furthermore, this thesis lays the foundation ground work for future research in advanced control techniques for flexible link manipulators. Potential future directions include experimental

validation of physical prototypes, integrating adaptive and learning-based controllers, and exploring energy-efficient control strategies.

This work provides a comprehensive framework for the robust control of flexible link manipulators, offering practical and scalable solutions for applications in industrial automation, space exploration, and precision tasks in surgical robotics.

Keywords: Flexible Link Manipulators, Distributed Parameter Systems, Assumed Mode Method, Sliding Mode Control, Functional Observer, Sliding Mode Observer.

सार

लचीले लिंक मैनिपुलेटर रोबोटिक मैनिपुलेटरों के एक ऐसे वर्ग का प्रतिनिधित्व करते हैं जो अपने हल्के डिज़ाइन के लिए जाने जाते हैं। लचीले लिंक मैनिपुलेटरों का हल्कापन उन्हें भूमि से लेकर अंतरिक्ष तक विभिन्न अनुप्रयोगों के लिए उपयुक्त बनाता है। हालाँकि, हल्केपन के गुण के साथ-साथ, एक अंतर्निहित लिंक लचीलापन भी होता है, जो नियंत्रण डिज़ाइन में गंभीर चुनौतियाँ प्रस्तुत करता है, जैसे कि युग्मित अरैखिक गतिकी और नोक पर कंपन, जो सटीक नोक स्थिति नियंत्रण को जटिल बनाता है। एकल लिंक लचीले मैनिपुलेटरों की तुलना में बहु-लिंक लचीले मैनिपुलेटरों का नियंत्रण परस्पर जुड़े लिंक युग्मनों के कारण और भी जटिल और गणना-संबंधी रूप से चुनौतीपूर्ण हो जाता है।

यह शोध प्रबंध एकल लिंक और बहु-लिंक लचीले मैनिपुलेटरों के लिए स्थिति नियंत्रण एल्गोरिदम डिज़ाइन करने पर केंद्रित है। अध्ययन यूलर-लैंग्रेज गति समीकरणों और कल्पित विधा विधि का उपयोग करके इन प्रणालियों के गणितीय मॉडलिंग से शुरू होता है, जो लिंक लचीलेपन की वितरित प्रकृति को पूरी तरह से पकड़ता है और नियंत्रण डिज़ाइन के लिए एक मजबूत आधार प्रदान करता है। एकल-लिंक लचीले मैनिपुलेटरों को प्रभावी ढंग से नियंत्रित करने के लिए एक कार्यात्मक प्रेक्षक-आधारित स्लाइडिंग मोड नियंत्रण डिज़ाइन प्रस्तावित किया गया है, जो अप्रतिरूपित गतिकी और विक्षोभ के प्रति सुदृढ़ता सुनिश्चित करता है। यह विधि सिस्टम अनिश्चितताओं को समायोजित करते हुए सटीक टिप पोजिशनिंग और कंपन शमन प्राप्त करती है। इसके अलावा, प्रेक्षकों के साथ पूरक इंटीग्रल स्लाइडिंग मोड और फास्ट टर्मिनल स्लाइडिंग मोड दृष्टिकोण जैसे उन्नत स्लाइडिंग मोड नियंत्रण एल्गोरिदम भी स्थिरता और अभिसरण के संदर्भ में अधिक ठोस परिणाम प्राप्त करने के लिए प्रस्तावित हैं। लचीली कड़ियों के युग्मन की गतिकी की अतिरिक्त जटिलता को दूर करने के लिए नियंत्रण ढाँचे को एक बहु-लिंक लचीले मैनिपुलेटर तक विस्तारित किया गया है। बहु-लिंक लचीले मैनिपुलेटर के लिए एक नवीन सुदृढ़ निश्चित समय इंटीग्रल स्लाइडिंग मोड नियंत्रण तकनीक प्रस्तावित की गई है, जो परस्पर संबद्ध कड़ियों पर गतिशील स्थिरता और सटीक टिप नियंत्रण सुनिश्चित करती है।

व्यापक संख्यात्मक सिमुलेशन, इस शोध प्रबंध में प्रस्तुत ट्रैकिंग और विनियमन समस्याओं में प्रस्तावित नियंत्रण एल्गोरिदम की सुदृढ़ता और प्रभावकारिता को प्रमाणित करते हैं। परिणाम एकल-लिंक से बहु-लिंक लचीले मैनिपुलेटर तक नियंत्रण डिज़ाइन विस्तार पर भी प्रकाश डालते हैं। यह शोध लचीली रोबोटिक प्रणालियों के लिए सुदृढ़ नियंत्रण एल्गोरिदम के उपयोग की हमारी समझ को आगे बढ़ाता है, जिससे जटिल, गतिशील सेटिंग्स के लिए विश्वसनीय समाधानों के डिज़ाइन में सहायता मिलती है। इसके अलावा, यह शोध प्रबंध लचीले लिंक मैनिपुलेटर्स के लिए उन्नत नियंत्रण तकनीकों में भविष्य के अनुसंधान की आधारशिला रखता है। संभावित भावी दिशाओं में भौतिक प्रोटोटाइप का प्रायोगिक सत्यापन, अनुकूली और अधिगम-आधारित नियंत्रकों का एकीकरण, और ऊर्जा-कुशल नियंत्रण रणनीतियों की खोज शामिल है।

यह कार्य लचीले लिंक मैनिपुलेटर्स के सुदृढ़ नियंत्रण के लिए एक व्यापक ढाँचा प्रदान करता है, जो औद्योगिक स्वचालन, अंतरिक्ष अन्वेषण और सर्जिकल रोबोटिक्स में परिशुद्धता कार्यों के अनुप्रयोगों के लिए व्यावहारिक और मापनीय समाधान प्रदान करता है।

Contents

Certificate	i
Acknowledgement	iii
Abstract	v
सार	vii
List of Figures	xiii
List of Tables	xv
List of Abbreviations	xvii
1 Introduction	1
1.1 Background and Motivation	1
1.2 Literature Review	2
1.2.1 Flexible Link Manipulator Model	2
1.2.2 Sliding Mode Control	4
1.2.3 Flexible Link Manipulator Control	5
1.3 Contributions	6
1.4 Organization of the Thesis	9
2 Preliminaries	11
2.1 Mathematical Modelling of Single Link Flexible Manipulator	11
2.2 Mathematical Modelling of Multi Link Flexible Manipulator	15
2.3 Sliding Mode Control	18
2.3.1 Sliding Mode for Continuous-time Systems	19
2.4 Integral Sliding Mode Control	21

2.5	Terminal Sliding Mode Control	22
2.6	Conclusion	24
3	Position Control using Functional Observer based Sliding Mode Control	25
3.1	Introduction	25
3.2	Problem Statement	26
3.3	Composite Control Design	26
3.3.1	Sliding Mode Control Design	26
3.3.2	Functional Observer	28
3.3.3	Proposed Functional Observer-based Sliding Mode Control	30
3.4	Simulation and Results	32
3.4.1	Regulation Problem	33
3.4.2	Tracking Problem	35
3.5	Conclusion	37
4	Functional Observer Based Output Feedback Sliding Mode Control Design using Balanced Truncated Reduced Order Model	39
4.1	Introduction	39
4.2	Problem Statement	40
4.3	Reduced Order Model of SLFM	40
4.4	Functional Observer Based Output Feedback Sliding Mode Control Design	40
4.4.1	Sliding Mode Control	41
4.4.2	Functional Observer	43
4.4.3	Composite Control Design	43
4.5	Results and Observations	44
4.5.1	Regulation	45
4.5.2	Tracking	48
4.6	Conclusion	51
5	Trajectory Tracking Control Design using an Observer based Integral Sliding Mode Control	53
5.1	Introduction	53
5.2	Problem Statement	53
5.3	Observer Based Integral Sliding Mode Control Design	54
5.3.1	Integral Sliding Mode Control	54

5.3.2	State Observer	57
5.3.3	Observer Based Integral Sliding Mode Composite Control Design	57
5.4	Results and Discussion	58
5.4.1	Simulation Results	58
5.4.2	Experimental Setup and Results	60
5.5	Conclusion	63
6	Finite Time Terminal Sliding Mode Control Design using Fixed Time Sliding Mode Observer	65
6.1	Introduction	65
6.2	Modelling of SLFM	65
6.2.1	Control Objective	67
6.2.2	Transformation to Controllable Canonical Form	67
6.3	Finite Time Composite Controller Design	68
6.3.1	Fast Terminal Sliding Mode Controller	68
6.3.2	Fixed Time Sliding Mode Observer	70
6.3.3	Composite Controller	72
6.4	Simulation Results	72
6.5	Conclusions	75
7	Position Control of Multi Link Flexible Manipulators	77
7.1	Introduction	77
7.2	State Space Model of Multi Link Flexible Manipulator	78
7.3	Problem Statement	79
7.4	Fixed-Time Integral Sliding Mode Control Design for Multi Link Flexible Manipulator	79
7.5	Simulation Results	81
7.5.1	Regulation	82
7.5.2	Tracking	84
7.6	Conclusion	86
8	Conclusions and Future Directions	87
8.1	Conclusions	87
8.2	Future Directions	88

Bibliography	91
Appendix	101
List of Publications	107
Bio-Data	109

List of Figures

1.1	Structure of the Thesis	10
2.1	Single Link Flexible Manipulator	11
2.2	Planar Two Link Flexible Manipulator	16
3.1	Output Plot for Regulation Problem	34
3.2	Sliding Function Plot for Regulation Problem	34
3.3	Control Input for Regulation Problem	35
3.4	Output Plot for Tracking Problem	36
3.5	Sliding Function Plot for Tracking Problem	36
3.6	Control Input for Tracking Problem	36
4.1	Workflow of Proposed Control Scheme	41
4.2	System Output Plot for Regulation Problem	45
4.3	Reduced Order System States Plot for Regulation Problem	47
4.4	Sliding Function Plot for Regulation Problem	47
4.5	Control Input Plot for Regulation Problem	48
4.6	System Output Plot for Tracking Problem	49
4.7	Reduced Order System States Plot for Tracking Problem	49
4.8	Sliding Function Plot for Tracking Problem	50
4.9	Control Input Plot for Tracking Problem	50
4.10	System Output Response	51
5.1	Output Plot for Tracking Problem	59
5.2	Sliding Function Plot for Tracking Problem	59
5.3	Control Input for Tracking Problem	60
5.4	Error Plot for Tracking Problem	60
5.5	Flexible Link Attached to Servomotor	61

5.6	Experimentation Setup	61
5.7	Tip Position Plot	62
5.8	Joint Angle Plot	62
5.9	Control Input Plot	62
6.1	Output Plots	73
6.2	States and Observer Comparison	73
6.3	States Plot for Original System	74
6.4	Sliding Variable Plots	74
6.5	Control Input Plot	75
7.1	Link-1 Joint Angle Plot	82
7.2	Link-2 Joint Angle Plot	83
7.3	Sliding Variable and Time Plot	83
7.4	Control Input and Time Plot	83
7.5	Vibratory Modes Versus Time Plot	84
7.6	Link-1 Joint Angle Plot Versus Time	84
7.7	Link-2 Joint Angle Plot Versus Time	85
7.8	Sliding Variable Versus Time Plot	85
7.9	Control Input Versus Time Plot	85
7.10	Vibratory Modes Versus Time Plot	86