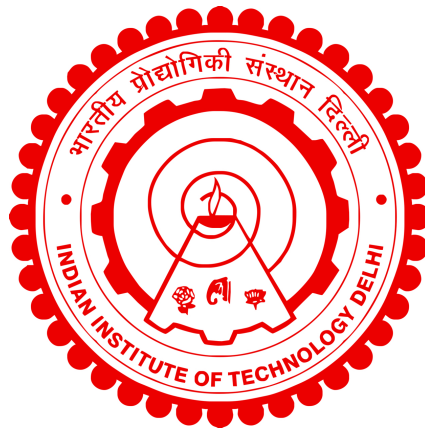


FLUID MODELING AND SIMULATIONS OF A FINITE SIZE UNMAGNETIZED PLASMA AND A FLOWING MAGNETIZED PLASMA

SUBHASISH BAG



DEPARTMENT OF PHYSICS

INDIAN INSTITUTE OF TECHNOLOGY DELHI

July 2025

© Indian Institute of Technology Delhi (IITD), New Delhi, 2025

Fluid Modeling and Simulations of a Finite Size Unmagnetized Plasma and a Flowing Magnetized Plasma

by

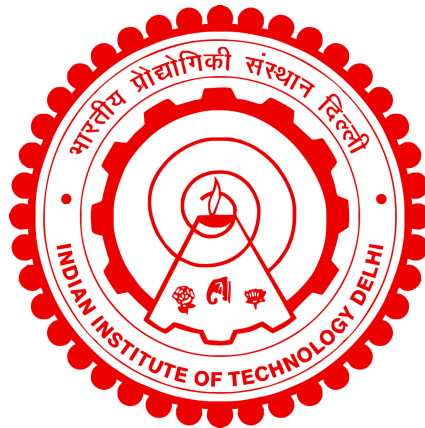
SUBHASISH BAG

Department of Physics

Submitted

in fulfillment of the requirements of the degree of Doctor of Philosophy

to the



INDIAN INSTITUTE OF TECHNOLOGY DELHI

July 2025

Dedicated to my parents

Certificate

This is to certify that the thesis titled “**Fluid Modeling and Simulations of a Finite Size Unmagnetized Plasma and a Flowing Magnetized Plasma**”, being submitted by **Subhasish Bag** to the Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy** in Physics, is a record of the original, bona fide research work carried out by him under my supervision and guidance. He has fulfilled the requirements for the submission of the thesis, which to the best of my knowledge has reached the required standard. The results contained in this thesis have not been submitted in part or in full to any other University or Institute for the award of any degree or diploma.

Prof. Vikrant Saxena

Thesis supervisor

Department of Physics,

Indian Institute of Technology Delhi

New Delhi-110016

Date: 21/7/2025

Acknowledgements

It begins with the term “Scintint” (an incorrect pronunciation of “scientist” in my first grade, which reflects my lifelong aspiration) and progresses one step closer to becoming a philosopher in Science. Firstly, I am grateful to my father, Samarendra Nath Bag; my mother, Archana Bag; my twin brother, Debasish Bag; and my two elder sisters, Sipra Bag and Soma Bag, for their unwavering support in this lifelong journey.

I express my profound gratitude to my supervisor, Prof. Vikrant Saxena, for his leadership and advice throughout this voyage, like how a magnetic field guides plasma particles during its expansion. During his supervision, I had the opportunity to collaborate with Prof. Ashish Ganguli, Prof. Amita Das, Prof. Ramesh Narayanan, and Prof. Debaprasad Sahu. He also allowed me to engage in an industry-funded project by the Indian Space Research Organization (ISRO) (not included in my thesis) with Dr. Ashis James from Liquid Propulsion Systems Centre (LPSC), the co-principal investigator of this research project. I would also like to thank Prof. P. K. Panigrahi (CQST/IISER Kolkata) for his constant support throughout my research journey.

I want to express my sincere gratitude to senior professors Prof. Ashish Ganguli and Prof. Amita Das for their guidance to my thesis (flowing magnetized plasma modeling and finite size plasma modeling respectively) and for developing my understanding of plasma physics. Each brief conversation during tea on various aspects of plasma physics, including fusion plasma, laser-plasma interaction, dusty plasma, and general physics, along with weekly lab presentations, allowed us to comprehend plasma physics comprehensively. The global meetings with various plasma physicists, organized by Prof. Amita Das, significantly enhanced our knowledge. The late-night discussion with Prof. Ramesh Narayanan regarding the experiment significantly enhanced my experimental understanding.

I want to thank my Student Research Committee (SRC) members, Prof. Amruta Mishra, Prof. Hitendra Kumar Malik, and Prof. Debaprasad Sahu, for their valuable suggestions and for assessing my semester progress reports during my presentation.

I extend my gratitude to Prof. Kazunori Takahashi of Tohoku University, Japan, for permitting me to visit his laboratory for two days and present my work to his group.

I thank the Indian Institute of Technology (IIT) Delhi for awarding me a five-year research fellowship. I wish to express my gratitude to IIT Delhi for the Research Scholar Travel Award (RSTA), the American Physical Society (APS), the Association of Asia Pacific Physical Societies (AAPPS), Council of Scientific & Industrial Research (CSIR), and Department of Science of Technology (DST) for the funding to attend the international conferences APS GEC 2022 in Sendai, Japan, AAPPS DPP 2023 in Nagoya, Japan, APS DPP 2024 in Atlanta, Georgia, USA, and IIS-2025 in Aix-en-Provence, France.

I extend my gratitude to my seniors Dr. Anshu Verma, Dr. Prashant Bansal, Dr. Arti Rawat, Dr. Preeti, Dr. Shweta Sharma, and Dr. Mahreen from the Experimental Plasma Physics Laboratory (PPL) of the Department of Energy Science and Engineering (DESE) for their guidance on experimental mechanisms pertaining to DC, RF, and Microwave (ECR) discharge experiments, as well as for fostering a conducive environment and engaging in Plasma Physics discussions during the early years of my PhD.

Pursuing a PhD is not merely an academic endeavor; it cultivates the ability to navigate life toward future aspirations by developing philosophical insights. I wish to thank Dr. Sooryansh and Dr. Rajni for their help discussing physics, maintaining a positive outlook, offering guidance to comprehend real-life scenarios, and managing and monitoring the challenges encountered throughout my PhD path. Furthermore, their academic recommendations are quite beneficial in achieving my objective. I also want to add Dr. Srimanta Maity for discussions on plasma physics and participation in sports and activities with us.

The designated workspace is the laboratory titled "Einstein Scholar Room" (MS310) of IIT Delhi, which will be marked with the names of my colleagues Imran Khan, Mamta Yadav, Laxman Goswami, and Priya Deshwal. The occasion was illuminated by the delicious food prepared by Imran and Mamta. I am also grateful to my laboratory colleagues Rohit Juneja, Trishul Dhalia, Subramanya Bhat K. N, and Aman Singh Katariya. Weekly Physics discussions, participation in conferences, tea/coffee following discussions, sports including swimming, and social outings for dining have been addressed with them. I want to thank Dr. Animesh Sharma for our productive physics chat. Our fresh junior, Md Yasir, assists in discussing and reviewing the thesis with minimal notice. I enumerate them on my journey.

This journey will be incomplete if I don't mention my PhD batchmates of IIT Delhi named Rahul, Nishant, Viresh, Lalit, Om, Vipul, Sunil, Sarvesh, Pallabi, Sourodeep, Ankit, Arjun, Ruchi, Diksha, Disha, Dipika, Manjari, Komal, Harshdhyan, Jitendra, Pithu, Shivangi, Yogesh.

Lastly, I possess a residence far from my primary abode, together with individuals from my IIT Delhi community: Rita Majumdar and Aakash Santra. They are really like family far from home. With them, I want to thank my other friends, Adrika Banerjee and Jyoti Rani Nath, for their continuous support during every ups and downs in my journey. From traveling and cooking to organizing Saraswati Puja and celebrating cultural events and musical performances with them consistently stimulates my intellect and gives me an open platform to extend my thoughts with peace of mind. This provides me with a sense of security following the research endeavor. With them, I will not forget to mention the name of my friend Arkajyoti De. I also want to thank Vikas Siwach, my MSc batchmate, for his unwavering support. Finally, I express my heartfelt gratitude to my beloved friend Srijan Mondal for his companionship in my journey.

This is one of the phases of life; there are so many persons, and names are countless. I sincerely acknowledge them for their support in my research journey.

Subhasish Bag

Abstract

This thesis examines, using fluid modeling, two distinct regimes of plasma dynamics, evolution of a finite size unmagnetized plasma and a flowing magnetized plasma. The former case, discussed in the first half of the thesis, is relevant when a plasma with finite extent and inhomogeneous density profiles is formed as a result of laser irradiation of a thin foil. Detailed investigations, based on one-dimensional fluid simulations, of the fundamental modes and dynamics of a finite size plasma reveal the onset of phase-mixing induced wave breaking process featuring sharp structures formation in the plasma density profile along with the generation of higher harmonics. It is also highlighted that the addition of diffusion suppresses the sharp structures, while an external forcing leads to local resonances in the finite-size plasma under consideration. It is also reported that wave breaking via phase mixing happens faster for the plasma with a sharper density gradient because of the generation of a strong field at the plasma edge. The case of flowing magnetized plasma, which has been investigated in the second half of the thesis, directly relates to the experiments pertaining to plasma-based electric propulsion as well as plasma processing. Considering an inhomogeneous magnetic field configuration, directly taken from an experiment, as an input for our modeling, a magnetic field based coordinate system has been developed. The steady-state fluid equations are then transformed into this new magnetic coordinate system and are simplified by using appropriate ordering. These simplified equations are integrated to obtain the profiles of flow variables in the magnetic coordinate system and are then transformed into the cylindrical coordinate system before comparing them against the experimental data. It is found that while all the flow profiles are in agreement with the experimental observations, the radial density profile does not show significant variation, unlike what was observed in the experiments. It can be reasoned that this difference in the radial variation of the ion density appears due to the absence of warm electrons in our model. The inclusion of warm electrons in the current model, however, needs further investigations and will be considered in the near future. Apart from that limitation the model presented is generic and can be adapted to any given cylindrically symmetric magnetic field configuration for exploring magnetized plasma dynamics.

सार

यह शोध प्रबंध, तरल मॉडलिंग का उपयोग करते हुए, प्लाज्मा गतिकी की दो अलग-अलग व्यवस्थाओं, एक परिमित आकार के अचुंबकीय प्लाज्मा और एक प्रवाहित चुंबकीय प्लाज्मा के विकास की जाँच करता है। प्रथम भाग में चर्चा की गई पूर्व स्थिति, तब प्रासंगिक होती है जब एक पतली पन्नी के लेजर विकिरण के परिणामस्वरूप परिमित विस्तार और विषम घनत्व प्रोफाइल वाला प्लाज्मा बनता है। एक-आयामी तरल सिमुलेशन पर आधारित, परिमित आकार के प्लाज्मा के मूलभूत मोड और गतिकी के विस्तृत अध्ययन से, प्रावस्था मिश्रण प्रेरित तरंग विखंडन प्रक्रिया की शुरुआत का पता चलता है, जिसमें प्लाज्मा घनत्व प्रोफाइल में तीक्ष्ण संरचनाओं का निर्माण और उच्च हार्मोनिक्स का निर्माण शामिल है। यह भी रेखांकित किया गया है कि विसरण के योग से तीक्ष्ण संरचनाएं दब जाती हैं, जबकि एक बाहरी बल विचाराधीन परिमित आकार के प्लाज्मा में स्थानीय अनुनादों को जन्म देता है। यह भी बताया गया है कि प्रावस्था मिश्रण के माध्यम से तरंग विखंडन, प्लाज्मा के किनारे पर एक प्रबल क्षेत्र के निर्माण के कारण, अधिक घनत्व प्रवणता वाले प्लाज्मा के लिए तेजी से होता है। प्रवाहित चुंबकीय प्लाज्मा का मामला, जिसकी शोध प्रबंध के दूसरे भाग में जाँच की गई है, सीधे प्लाज्मा-आधारित विद्युत प्रणोदन और प्लाज्मा प्रसंस्करण से संबंधित प्रयोगों से संबंधित है। एक प्रयोग से सीधे लिए गए एक विषम चुंबकीय क्षेत्र विन्यास को हमारे मॉडलिंग के लिए एक इनपुट के रूप में मानते हुए, एक चुंबकीय क्षेत्र आधारित निर्देशांक प्रणाली विकसित की गई है। स्थिर-अवस्था तरल समीकरणों को फिर इस नई चुंबकीय निर्देशांक प्रणाली में रूपांतरित किया जाता है और उपयुक्त क्रम का उपयोग करके सरलीकृत किया जाता है। इन सरलीकृत समीकरणों को चुंबकीय निर्देशांक प्रणाली में प्रवाह चरों की रूपरेखा प्राप्त करने के लिए एकीकृत किया जाता है और फिर प्रयोगात्मक आँकड़ों से तुलना करने से पहले बेलनाकार निर्देशांक प्रणाली में रूपांतरित किया जाता है। यह पाया गया है कि यद्यपि सभी प्रवाह रूपरेखाएँ प्रयोगात्मक प्रेक्षणों के अनुरूप हैं, रेडियल घनत्व रूपरेखा में प्रयोगों में देखी गई भिन्नता के विपरीत कोई महत्वपूर्ण परिवर्तन नहीं दिखाई देता है। यह तर्क दिया जा सकता है कि आयन घनत्व की रेडियल भिन्नता में यह अंतर हमारे मॉडल में गर्म इलेक्ट्रॉनों की अनुपस्थिति के कारण दिखाई देता है। हालाँकि, वर्तमान मॉडल में गर्म इलेक्ट्रॉनों को शामिल करने के लिए और अधिक जाँच की आवश्यकता है और निकट भविष्य में इस पर विचार किया जाएगा। उस सीमा के अलावा, प्रस्तुत मॉडल सामान्य है और इसे चुंबकीय प्लाज्मा गतिकी की खोज के लिए किसी भी बेलनाकार सममित चुंबकीय क्षेत्र विन्यास में अनुकूलित किया जा सकता है।

Contents

Certificate	i
Acknowledgements	ii
Abstract	v
सार	vi
List of Figures	xi
List of Tables	xviii
1 Introduction	1
1.1 Background and Motivation	1
1.1.1 Finite size plasmas	4
1.1.2 Plasma expansion in a diverging magnetic field	4
1.2 Review of the previous work	8
1.2.1 Dynamics of finite size plasmas	8
1.2.2 Phase mixing induced wave breaking	9
1.2.3 Modeling of flowing plasma guided by magnetic field in the ap- plication of plasma-based electric propulsion	11
1.3 Objective and Organization of the thesis	14
1.3.1 Chapter 2: Study of Finite Size Plasma via Fluid Simulation . .	15
1.3.2 Chapter 3: Phase Mixing in Finite Size Plasma	16
1.3.3 Chapter 4: Flowing Magnetised Plasma: Theoretical Model and Numerical Approach	16

1.3.4	Chapter 5: Flowing Magnetised Plasma: Results for a Small Volume Experiment	17
1.3.5	Chapter 6: Conclusion and Future Work	18
I	<i>Modeling and Simulations of a Finite Size Unmagnetized Plasma</i>	19
2	Study of Finite Size Plasma via Fluid Simulation	20
2.1	Introduction	20
2.2	Two-Fluid Model of Plasma	21
2.2.1	Numerical Methodology	23
2.3	Two-Fluid Simulation for Finite Size Plasma	27
2.3.1	Modeling and Dynamics of Finite Size Plasma	27
2.3.2	Effect of Additional Diffusion on Plasma Evolution	36
2.3.3	Effect of External Forcing	41
2.3.4	Reversibility of External Field:	45
2.4	Conclusion	46
3	Phase Mixing in Finite Size Plasma	48
3.1	Introduction:	48
3.2	Dynamical Evolution of Perturbed Plasmas	49
3.3	Nonlinear evolution of a periodic plasma	57
3.3.1	Sinusoidal perturbation in electron density with initially homogeneous ion density	58
3.3.2	Sinusoidal perturbation in the densities of electron and (mobile) ion fluids	61
3.3.3	Electron plasma oscillations in an inhomogeneous static ion background	62
3.4	Nonlinear evolution of a finite-size plasma	63
3.5	Fluid Simulation of periodic and finite size plasma	66
3.6	Conclusions	70
	Appendix	73
3.A	Correction of δn_d corrected up to third orders:	73

II	<i>Modeling of a Flowing Magnetized Plasma and Results for a Small Volume Experiment</i>	74
4	Flowing Magnetised Plasma: Theoretical Model and Numerical Approach	75
4.1	Introduction	75
4.2	Theoretical Model	76
4.2.1	Stationary Model Equations for a flowing magnetised plasma .	76
4.2.2	Transformations to Magnetic coordinates	78
4.2.3	Flow in the Magnetic Geometry	80
4.2.4	Flow Governed by the Magnetic Field	87
4.3	Methodology to Construct the Magnetic Grid	96
4.3.1	Construction of initial grid points for small volume plasma experiment	96
4.3.2	Algorithm for constructing the magnetic grid	105
4.4	Grid configuration in experimental and magnetic geometries	109
	Appendix	112
4.A	Mathematical Formulations in Coordinate Transformations	112
4.B	Axial profile of a_1	117
4.C	Axial profile of a_2	120
5	Flowing Magnetised Plasma: Results for a Small Volume Experiment	122
5.1	Introduction	122
5.2	Initial Conditions	124
5.3	Results and Discussion	125
5.3.1	Flow profiles in magnetic coordinates	125
5.3.2	Transformations from magnetic coordinates to experimental coordinates	134
5.3.3	Flow profiles in experimental(cylindrical) coordinates	138
5.3.4	Flux, Thrust, and Energy Calculations	146
5.3.5	Comparison of model and experimental radial measurements . .	151
5.4	Conclusions	153
	Appendix	155

5.A	Calculation of electron thrust due to the electron thermal pressure . . .	155
6	Summary, Conclusions, and Future Directions	156
6.1	Summary and Conclusions	156
6.1.1	Part A: Modeling and Simulations of a Finite Size Unmagnetized Plasma.	156
6.1.2	Part B: Modeling of a Flowing Magnetized Plasma and results for a small volume experiment.	157
6.2	Future Directions	158
6.2.1	Thermal effects on the evolution of finite-size plasma	158
6.2.2	Extention of flow model for small volume plasma experiment . .	159
6.2.3	Modeling of Energy transfer from electrons to ions	159
	Bibliography	160
	List of Publications	177
	Bio-data	183

List of Figures

1.1	Overview of Plasma Simulation	2
1.2	In (a), the schematic of the Small Volume Plasma System (SVPS) with Compact ECR Plasma Source (CEPS) has been shown. In (b), the axial and 2D profile of the magnetic field have been shown [1]. . .	6
1.3	The Overview of the Thesis	14
2.1	Total Energy for the two cases where E_p , E_k , E_{tot} are the Potential Energy, Kinetic Energy (total), and Total Energy, respectively.	26
2.2	Initial Density profiles. (2.2a) has the smoother edge and (2.2b) sharper edge.	26
2.3	Initial electric field profiles. (2.3a) has the smoother edge and (2.3b) sharper edge.	27
2.4	Density Evolution of the case: figure-(2.2a).	28
2.5	Density Evolution of the case: figure-(2.2b).	29
2.6	Density Evolution at the central region of the case: figure-(2.2a).	30
2.7	Density Evolution at the central region of the case: figure-(2.2b).	31
2.8	Total Energy for two cases (figures-2.2a and 2.2b) where E_p , E_{ke} , E_t are the Potential Energy, Electron Kinetic Energy, and Total Energy, respectively.	31
2.9	Total energy with different dx for the smooth density case: figure-(2.2a) and the sharp density case: figure-(2.2b).	32
2.10	Density of electron corresponding t=24 and 26 for smooth profile case with dx= 0.01 and 0.001.	32
2.11	Density of electron corresponding t=7 and 8 for sharp profile case with dx= 0.01 and 0.001.	33
2.12	Spatial FFT for the smooth and sharp cases at different times.	34
2.13	Temporal FFT for the smooth and sharp cases at different locations.	34

2.14	Time of Energy Damping (τ) with density scale length at the profile edge.	36
2.15	Electron density Evolution for $\eta = 0.001$	36
2.16	Electron density Evolution for $\eta = 0.005$	37
2.17	Electron density Evolution for $\eta = 0.01$	37
2.18	Electron density Evolution for different η at $t=20$	37
2.19	Electron density Evolution for different η at $t=40$	38
2.20	Total energy with viscous effect for different η	39
2.21	Mean Square Displacement (MSD) of electron and ion fluid.	39
2.22	Mean Square Displacement D of electron (D_e) and ion (D_i) fluid with the external diffusion constant η	40
2.23	Electron density evolution for the external electric field $E = 0.01 \cos(t)$	42
2.24	Electron density evolution for the external electric field $E = 0.01 \cos(0.5t)$	42
2.25	Electron density evolution for the external electric field $E = 0.01 \cos(0.01t)$	43
2.26	Electron density evolution for the external electric field $E = \cos(0.01t)$, $E = \cos(0.5t)$, $E = \cos(t)$	43
2.27	Externally applied electric field $E = 0.01 \cos(t)$ for a time duration of $t=0$ to 10 unit in this subfigure (2.27a) and Total Energy for this case in the subfigure (2.27b) where E_p , E_{ke} , E_t are the Potential Energy, Electron Kinetic Energy, and Total Energy, respectively.	44
2.28	$\langle J.E \rangle$ for electron and ion in figures (2.28a) and (2.28b) respectively.	45
2.29	Generation of sharp structures at the edge of the plasma profile in the case of the short termed externally driven system.	46
3.1	Evolution of the analytical solution of δn_d showing the generation of higher order modes around the phase mixing damping time in (3.1a) for the case of periodically perturbed electron with initially homogeneous mobile ions. In fig (3.1b), the k-spectra shows the growth in higher order modes at the expense of initially excited modes.	59
3.2	Evolution of the analytical solution of δn_d showing the generation of higher order modes around the phase mixing time in (3.2a) for the case of periodically perturbed electrons and ions. In fig (3.2b), the k-spectra shows the energy transfer to higher-order modes.	62

3.3	Evolution of the analytical solution of δn_d showing the generation of higher order modes around the phase mixing damping time in (3.3a) for the case of a finite-size plasma. In fig (3.3b), the k-spectra shows the growth of the higher order modes.	65
3.4	Total Energy for initially periodic electron density ($n_e(x, 0) = 1 + 0.3 \cos(kx)$) having mobile ($\Delta = \frac{m_e}{m_i} = 0.001$) homogeneous ion background where E_p , E_{ke} , E_t are the Potential Energy, Electron Kinetic Energy, and Total Energy, respectively. Damping starts at $\approx 64 \omega_p^{-1}$	68
3.5	Comparison of the evolution of δn_d from analytical modelling and simulation when we have initial homogeneous finite ion mass.	68
3.6	Total Energy for initially periodic electron density ($n_e(x, 0) = 1 + \delta \cos(kx)$) having immobile ($\Delta = \frac{m_e}{m_i} = 0$) homogeneous ion background where E_p , E_{ke} , E_t are the Potential Energy, Electron Kinetic Energy, and Total Energy, respectively. Here, δ is 0.3 for figure (3.6a) and 0.8 for figure (3.6b). No wave-breaking happens for $\delta = 0.3$ (as $\delta < 0.5$), and breaking has been observed within a period of oscillation for $\delta = 0.8$ (as $\delta > 0.5$).	69
3.7	Total Energy for initially periodic electron density ($n_e(x, 0) = 1 + \delta_e \cos(kx)$) having mobile ($\Delta = \frac{m_e}{m_i} = 0.01$) inhomogeneous ion ($n_e(x, 0) = 1 + \delta_i \cos(kx)$) where E_p , E_{ke} , E_t are the Potential Energy, Electron Kinetic Energy, and Total Energy, respectively. Here, δ_e and δ_i are 0.3 and 0.003 respectively. The predicted wave-breaking time from the energy damping is $\approx 30 \omega_p^{-1}$	70
3.8	Total Energy for the case of a finite size plasma having mobile ($\Delta = \frac{m_e}{m_i} = 0.001$) ions where E_p , E_{ke} , E_t are the Potential Energy, Electron Kinetic Energy, and Total Energy, respectively. Damping sets in at around $\approx 13 \omega_p^{-1}$	71
4.2.1	Flow variables (v_r, v_z) in Cylindrical Geometry. Flow variables ($v_{ }, v_{\perp}$) in Magnetic Geometry. Change of Flow variables on magnetic field lines in the expansion region	77
4.2.2	Unit vectors in Magnetic coordinate system	79
4.2.3	Electrons in magnetic flux tube	86
4.3.1	The figure for the measurement of Δs for a given fraction error.	98

4.3.2	The figure for the measurement of Δt for a given fraction error. . . .	99
4.3.3	Small Volume Plasma System (SVPS) experiment	100
4.3.4	Minimum Δt with respect to Z and R axes	101
4.3.5	Minimum Δt with respect to Z and R axes	102
4.3.6	Minimum Δt with respect to Z and R axes	103
4.3.7	Finding the new unknown grid point $(i + 1, j + 1)$, when we have three known coordinates (i, j) , $(i + 1, j)$, $(i, j + 1)$	105
4.3.8	Experimental geometry in $s - t$ coordinate	108
4.3.9	Experimental geometry in $r - z$ coordinate	108
4.3.10	The Perpendicular $s - t$ Lines	109
4.3.11	Experimental symmetric section of the chamber	109
4.4.1	Two dimensional b_r profile	110
4.4.2	Two dimensional b_z profile	110
4.4.3	Two dimensional B profile	110
5.3.1	Constant s and t lines	125
5.3.2	Axial profiles of $nU_{ }/B$ at two different pressures- fig (5.3.2a) for 0.5 mTorr and fig (5.3.2b) for 5.0 mTorr case.	126
5.3.3	2D Density (n) profile (5.0 mTorr) in $s - t$ coordinate- fig (5.3.3a) for 0.5 mTorr and fig (5.3.3b) for 5.0 mTorr case.	127
5.3.4	Axial Density (n) profile- fig (5.3.4a) for 0.5 mTorr and fig (5.3.4b) for 5.0 mTorr case.	127
5.3.5	Density n vs s - fig (5.3.5a) for 0.5 mTorr and fig (5.3.5b) for 5.0 mTorr case.	127
5.3.6	Density n vs t - fig (5.3.6a) for 0.5 mTorr and fig (5.3.6b) for 5.0 mTorr case.	128
5.3.7	2D Potential (ϕ) profile - fig (5.3.7a) for 0.5 mTorr and fig (5.3.7b) for 5.0 mTorr case.	129
5.3.8	Axial Potential (ϕ) profile - fig (5.3.8a) for 0.5 mTorr and fig (5.3.8b) for 5.0 mTorr case.	129
5.3.9	Potential ϕ vs s - fig (5.3.9a) for 0.5 mTorr and fig (5.3.9b) for 5.0 mTorr case.	130
5.3.10	Potential ϕ vs t - fig (5.3.10a) for 0.5 mTorr and fig (5.3.10b) for 5.0 mTorr case.	130

5.3.11 2D Velocity ($U_{ }$) profile - fig (5.3.11a) for 0.5 mTorr and fig (5.3.11b) for 5.0 mTorr case.	131
5.3.12 Parallel flow velocity $U_{ }$ vs s - fig (5.3.12a) for 0.5 mTorr and fig (5.3.12b) for 5.0 mTorr case.	131
5.3.13 Parallel flow velocity $U_{ }$ vs s - fig (5.3.13a) for 0.5 mTorr and fig (5.3.13b) for 5.0 mTorr case.	132
5.3.14 n/B scaling at constant t - fig (5.3.14a) for 0.5 mTorr and fig (5.3.14b) for 5.0 mTorr case.	132
5.3.15 2D Velocity ($U_{\perp}$) profile - fig (5.3.15a) for 0.5 mTorr and fig (5.3.15b) for 5.0 mTorr case.	132
5.3.16 Perpendicular flow velocity $U_{\perp}$ vs s - fig (5.3.16a) for 0.5 mTorr and fig (5.3.16b) for 5.0 mTorr case.	133
5.3.17 Perpendicular flow velocity $U_{\perp}$ vs t - fig (5.3.17a) for 0.5 mTorr and fig (5.3.17b) for 5.0 mTorr case.	133
5.3.18 2D Velocity $ U_{\theta} $ profile - fig (5.3.18a) for 0.5 mTorr and fig (5.3.18b) for 5.0 mTorr case.	134
5.3.19 U_{θ} vs s - fig (5.3.19a) for 0.5 mTorr and fig (5.3.19b) for 5.0 mTorr case.	134
5.3.20 U_{θ} vs t - fig (5.3.20a) for 0.5 mTorr and fig (5.3.20b) for 5.0 mTorr case.	135
5.3.21 U_{θ} vs s - fig (5.3.21a) for 0.5 mTorr and fig (5.3.21b) for 5.0 mTorr case.	135
5.3.22 U_{θ} vs t - fig (5.3.22a) for 0.5 mTorr and fig (5.3.22b) for 5.0 mTorr case.	136
5.3.23 Constant r and z lines	136
5.3.24 b_r at constant r and z lines	137
5.3.25 b_z at constant r and z lines	137
5.3.26 2D Density (n) profile - fig (5.3.26a) for 0.5 mTorr and fig (5.3.26b) for 5.0 mTorr case.	139
5.3.27 n vs r - fig (5.3.27a) for 0.5 mTorr and fig (5.3.27b) for 5.0 mTorr case.	139
5.3.28 n vs z - fig (5.3.28a) for 0.5 mTorr and fig (5.3.28b) for 5.0 mTorr case.	139
5.3.29 2D Potential (ϕ) profile - fig (5.3.29a) for 0.5 mTorr and fig (5.3.29b) for 5.0 mTorr case.	140
5.3.30 ϕ vs r - fig (5.3.30a) for 0.5 mTorr and fig (5.3.30b) for 5.0 mTorr case.	141
5.3.31 ϕ vs z - fig (5.3.31a) for 0.5 mTorr and fig (5.3.31b) for 5.0 mTorr case.	141

5.3.32 2D Velocity (U_r) profile - fig (5.3.32a) for 0.5 mTorr and fig (5.3.32b) for 5.0 mTorr case.	142
5.3.33 U_r vs r - fig (5.3.33a) for 0.5 mTorr and fig (5.3.33b) for 5.0 mTorr case.	142
5.3.34 U_r vs z - fig (5.3.34a) for 0.5 mTorr and fig (5.3.34b) for 5.0 mTorr case.	142
5.3.35 2D Velocity (U_z) profile - fig (5.3.35a) for 0.5 mTorr and fig (5.3.35b) for 5.0 mTorr case.	143
5.3.36 U_z vs r - fig (5.3.36a) for 0.5 mTorr and fig (5.3.36b) for 5.0 mTorr case.	143
5.3.37 U_z vs z - fig (5.3.37a) for 0.5 mTorr and fig (5.3.37b) for 5.0 mTorr case.	143
5.3.38 2D Velocity $ U_\theta $ profile - fig (5.3.38a) for 0.5 mTorr and fig (5.3.38b) for 5.0 mTorr case.	144
5.3.39 U_θ vs r - fig (5.3.39a) for 0.5 mTorr and fig (5.3.39b) for 5.0 mTorr case.	144
5.3.40 U_θ vs z - fig (5.3.40a) for 0.5 mTorr and fig (5.3.40b) for 5.0 mTorr case.	145
5.3.41 ν_θ vs r - fig (5.3.41a) for 0.5 mTorr and fig (5.3.41b) for 5.0 mTorr case.	145
5.3.42 ν_θ vs z - fig (5.3.42a) for 0.5 mTorr and fig (5.3.42b) for 5.0 mTorr case.	145
5.3.43 Radial energy profiles- fig (5.3.43a) for 0.5 mTorr and fig (5.3.43b) for 5.0 mTorr case.	146
5.3.44 Axial energy profiles- fig (5.3.44a) for 0.5 mTorr and fig (5.3.44b) for 5.0 mTorr case.	146
5.3.45 Flux profiles- fig (5.3.45a) for 0.5 mTorr and fig (5.3.45b) for 5.0 mTorr case.	147
5.3.46 Flux profiles- fig (5.3.46a) for 0.5 mTorr and fig (5.3.46b) for 5.0 mTorr case.	147
5.3.47 Flow energy (nU_r^2)- fig (5.3.47a) for 0.5 mTorr and fig (5.3.47b) for 5.0 mTorr case.	147
5.3.48 Flow energy (nU_z^2)- fig (5.3.48a) for 0.5 mTorr and fig (5.3.48b) for 5.0 mTorr case.	148
5.3.49 Interpolation of ion thrust at the plasma source section ($z = 0.088\text{m}$)	150

- 5.3.50 Comparison of radial profiles of the flow model and the experimental measurements for 0.5 mTorr (a,b,c) and 5 mTorr (d,e,f) pressure cases at $z = 0.281$ m constant line. n_b and n_w are the experimentally measured densities of bulk and warm electron populations, whereas n is the flow modeling density result. $\bar{n}_{b0}$ is the ratio constant used to get unity n_b and n_w densities. Similarly, the corresponding temperatures (T_{be}, T_{we}, T_e) and potentials (ϕ_b, ϕ) have been shown. 152

List of Tables

2.1	Normalized variables-I	22
3.1	Normalized variables-II	50
4.2.1	Normalized variables-III	83
4.2.2	Ordering of variables	85
4.3.1	Maximum allowable Δt , Δs for 0.1% fractional error	101
5.2.1	Table of Initial Parameters	124
5.3.1	Table for the ion and electron thrust estimation at different z locations for 0.5 mTorr pressure case.	149
5.3.2	Table for the ion and electron thrust estimation at different z locations for 5.0 mTorr pressure case.	149
5.3.3	Table of Radial dependency of thrust estimation at different z locations for 0.5 mTorr pressure case.	150
5.3.4	Estimation of power deposition by ions at different z locations.	151
5.3.5	Table of Radial dependency of power deposition by ions at different z locations for 0.5 mTorr pressure case.	151