

**MODIFIED HARMONICS METHOD  
FOR ANALYSIS AND DESIGN  
OF  
SKEW GIRDER BRIDGES**

**A THESIS**

*submitted in fulfilment of the  
requirements for the award of the degree*

*of*

**DOCTOR OF PHILOSOPHY**

*in*

**CIVIL ENGINEERING**

*by*

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**INDIAN INSTITUTE OF TECHNOLOGY  
DELHI, INDIA**

**1982**

## CERTIFICATE

This is to certify that the thesis entitled, 'MODIFIED HARMONICS METHOD FOR ANALYSIS AND DESIGN OF SKEW GIRDER BRIDGES' being submitted by Sri Jagdish Prasad to the Indian Institute of Technology, Delhi for the award of the degree of Doctor of Philosophy in Civil Engineering, is a record of bonafide research work carried out by him. Sri Jagdish Prasad has worked under my guidance and supervision and has fulfilled the requirements for the submission of this thesis, which, to my knowledge, has reached the requisite standard.

The matter embodied in this thesis has not been submitted in part or in full, to any other University or Institute for the award of any degree or diploma.



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## ACKNOWLEDGEMENT

The author sincerely expresses his gratitude to Dr.C.S. Surana, Professor, Department of Civil Engineering under whose overall guidance and supervision the research work reported in this thesis was carried out. Through his critical comments, constructive suggestions and vast experience, Dr. Surana induced a high sense of scientific temper in the author and got him totally involved in the research work being carried out. The author feels deeply indebted to his guide for having had a scientific training in the purposeful engineering research work carried out and recorded in this thesis.

Dr. B.M. Ahuja, Professor, Civil Engineering Department, evinced keen interest and made many constructive suggestions throughout the progress of the experimental part of the research work. The author records herein his deep sense of gratitude to Dr. Ahuja for his invaluable suggestions and sharing his vast experience.

The helpful attitude and sincere efforts put in by the staff of the Structural Engineering Laboratory and Department Workshop for the successful accomplishment of the experimental work is gratefully acknowledged.

The helpful hands extended by many of the persons around the author is keenly felt and thankfully acknowledged.

  
(Jagdish Prasad)

## SYNOPSIS

Many a time the topography of a proposed bridge site makes it inevitable to align the bridge at a skew angle. Such cases arise quite often in mountainous regions. It is, therefore imperative to thoroughly understand the structural behaviour of skew girder-bridges. It would also be pertinent to examine a generalized method of girder-bridge analysis for its suitability in design office without resorting to the use of digital computer. Harmonics Method of girder-bridge analysis has been chosen for this purpose which eminently lends itself to producing design aids to be used in design office. The study would serve the twofold purpose of saving in respect of time and cost of analysis; and would also eliminate the need of a digital computer which may not always be readily available in design offices.

The various methods of bridge superstructure analysis commonly adopted are as follows

- (1) Finite Element Method
- (2) Stiffness Matrix Method
- (3) Finite Strip Method
- (4) Orthotropic Plate Theory; and
- (5) Harmonics Method

Finite Element Method is a highly versatile method of structural analysis with capability to tackle any structure having arbitrary physical constraints and configuration of applied loading. The method, however, needs careful preparation of large amounts of data required for the computer programme; and

convergence study of the solutions for every new structure(5) . This makes the method unsuitable for an actual bridge design wherein many trial bridge sections are required to be quickly analyzed for various loading cases. Stiffness Matrix and Finite Strip Methods, too, suffer from similar difficulties. However, Stiffness Matrix Method takes less computation time primarily due to smaller amount of data involved. It is significant to note that all these methods become total non-performers in the absence of high speed digital computer. Orthotropic Plate Theory provides analytical solutions in comparatively limited and simple cases of loading and physical constraints. Harmonics Method as originally developed by Hendry and Jaeger (1) and subsequently modified and extended by Surana and others(2,3) to incorporate torsional stiffness provides a general method of girder-bridge analysis without encountering the above mentioned difficulties. Harmonics Method has yet another advantage in its inherent capability to completely identify a girder-bridge by a certain combination of non-dimensional structural parameters. These parameters help visualize the structural behaviour due to change in bridge dimensions which in turn help in decision making about the trial sections.

In this thesis an indepth study of the Harmonics Method of analysis for skew girder-bridges is made and the limitations are brought-out. The scope of the method is extended and refinements are incorporated to further increase its applicability in extreme cases of bridge-loadings and bridge-element

configurations. For ready use in design offices, bending moment Design Coefficients for various combinations of the structural parameters have been developed and suitably presented. For these studies, the results obtained by the Harmonics Method have been compared with the ones obtained by Stiffness Method and also with the experimental observations made on carefully chosen bridge models.

With a view to understanding the structural behaviour of skew girder-bridges due to changes in certain bridge parameters, a few bridge models have been studied using Stiffness Matrix Method. The effects of change in skew angle on moments in the girders, support reactions, transverse moments in the slab and rotations of girders (both along the length of the girders as well as normal to them) have been studied. Cross-girders are the main agents of load distribution amongst the longitudinal girders. The effects of the orientations of the cross-girders, either parallel to the supports or perpendicular to the longitudinal girders have been studied with a view to achieving an efficient system. Wide skew bridges have been analyzed by Stiffness Matrix Method and applicability of the Harmonics Method for such bridges studied.

Harmonics Method developed so far has been used to analyze only simply supported single span girder-bridges. In the present work, the method has been extended and applied to analyze continuous two-span girder-bridges. The results on continuous bridges are found to compare well with the results obtained by using Stiffness Matrix Method.

Harmonics Method due to its intrinsic property spreads out the effects of imposed concentrated load and hence exhibits some deviation in bending moment values from those obtained by Stiffness Method. This is particularly so when only one or two concentrated loads act on a girder. Although this type of loading does not represent real bridge loading cases, a procedure has been evolved and presented in this thesis to reasonably improve bending moment values in the vicinity of imposed concentrated loads.

As has been stated earlier, Harmonics Method Completely identifies a girder-bridge by a certain combination of the structural parameters defined in the thesis. Taking advantage of this capability of the method, bending moment Design Coefficients have been computed for different combinations of these parameters covering real-life bridges. These bending moment Design Coefficients for four-girder bridges have been presented in tabular form in the thesis. These coefficients can readily be used in a design office to analyze four-girder bridges or a multi-girder bridge after converting it into an equivalent four-girder bridge(1), without any recourse to a digital computer. A four-girder bridge has been analyzed as an example using the bending moment Design Coefficients presented in the thesis. Bending moment Design Coefficients for three-girder bridges have already been presented in earlier work(3,4).

To assess the validity of the method of analysis present in the thesis, experiments for structural behaviour have been

conducted on carefully chosen bridge models. The material chosen for the bridge models conforms in its properties to the assumptions made in the theory. Reinforced concrete was considered unsuitable for not being truly homogeneous, linearly elastic and creep-free besides suffering from micro-cracking, shrinkage and bond-slip problems. An especially developed Sand-Araldite system has been found to satisfactorily meet all the requirements incorporated in the assumptions made in the theory. Sand is obtained from finely crushed quartz stone after processing it through a set of sieves. Araldite used is a two-component system consisting of two parts of Araldite GY-257 and one part of Hardener HY-840 by weight. This was supplied by CIBA-GEIGY of India Ltd. The Sand-Araldite system was tested in different mix proportions for workability, compressive and tensile strengths, stress-strain linearity and creep phenomenon. The Sand-Araldite mix in proportion of 8:1 by weight was found to have good workability, high compressive and tensile strengths and linear stress-strain relationship. It was also found to be free from creep effect in the elastic stress range. The material is homogeneous and can easily be cast monolithically into any shape and size. High tensile strength of the material eliminates the need for steel reinforcement. Young's Modulus of Elasticity and Poisson's ratio of the material are similar to those of concrete.

Five Sand-Araldite bridge models each differing in skew angle and/or number of longitudinal girders with or without cross-girders have been tested for various structural behaviour. The bridge structural parameters of these models vary in such a

manner that they lie in the practical range of skew bridges. These bridge models have been investigated for girder deflection rotations along the length of the girders and also normal to it. strains in the girder top and bottom fibres, strains in the deck slab at various locations, and support reactions. Girder deflections and rotations were measured using magnetic base dial gauges. Strain gauges in the slab were affixed in the orthogonal directions in order to incorporate the Poisson's effects in the computations. Support reactions have been measured using diaphragm type load cells especially designed and fabricated for the purpose. Load cells were designed for the load capacity of 50 kg with 15-micron central deflection at full load. The system is self temperature compensated and free from hysteresis effects.

The theoretical and experimental studies carried out on the bridge models vividly bring out the fact that Harmonics Method as presented in the thesis works satisfactorily and provides a powerful method for girder-bridge analysis. The method by virtue of its inherent capability to produce design aids in tabular or graphical form is highly suitable for use in design office effecting a lot of reduction in time and cost of analysis of girder bridges.

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