

SIMILARITY METHODS IN FLUID MECHANICS

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**A THESIS ON
SIMILARITY METHODS IN FLUID MECHANICS**

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DEDICATED TO MY DEPARTED PARENTS

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