SIMULATION OF NONLINEAR DYNAMIC FINITE STRAIN CONTACT PROBLEMS WITH SPECIAL APPLICATION TO HIGH VELOCITY IMPACT

By

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DEDICATED TO MY

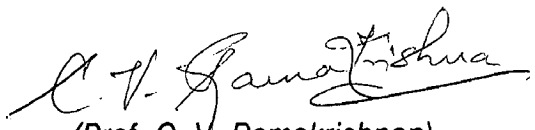
FATHER *SHRI G. SATYANARAYANA*

MOTHER *SMT. G. SUNDARAMMA*

CERTIFICATE

This is to certify that the thesis entitled ***'Simulation of Nonlinear Dynamic Finite Strain Contact Problems with Special Application to High Velocity Impact'*** *being submitted by* ***Mr. G. V. V. Ravi Kumar*** *to the* ***Indian Institute of Technology, Delhi*** *for the award of the Degree of Doctor of Philosophy in Applied Mechanics is a record of bonafide research work carried by him under my supervision and guidance. The thesis work, in my opinion, has reached the requisite standard fulfilling the requirement for the Doctor of Philosophy Degree.*

The results contained in this thesis have not been submitted in part or in full, to any other University or Institute for the award of any degree or diploma.



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ABSTRACT

An impact simulation system based on rate independent displacement based finite element formulation which can simulate low/medium/high velocities has been presented. The development of this system at various stages has been discussed after a brief introduction on high velocity impact phenomena. In the first stage the theory, computer implementation and validation of material nonlinear dynamic problems have been discussed. In the second stage, a contact algorithm based on plasticity theory of friction and joint elements has been developed. This algorithm has a good constitutive basis and can simulate stick, slip and debonding modes of contact in both static and dynamic environment. This algorithm has been applied to a class of practical structural dynamic problem for the estimation of root damping in turbine blades. The effect of joint element stiffness parameters on root damping has been studied and suitable values of stiffness parameters have been suggested. Some parametric studies have also been carried out for the root damping.

In the next stage, the material nonlinear dynamic model has been extended to simulate large deformations which occur in high velocity impact. Incremental Jaumann stresses and incremental deformation strains have been used in the model. This model uses an updated Lagrangian formulation. An elegant rezoning procedure has been developed in the context of impact simulation. This uses a semi-automated mapped mesh generation along with a simple mapping procedure for the transformation field variables from old mesh to new mesh. This algorithm includes checks for mass, momentum and energy conservation between old and new meshes during rezoning.

The contact algorithm developed for structural dynamic problems has been modified and coupled with the large deformation wave propagation model. Shock pressures have been computed using Mie-Gruneisen equation of state. The impact simulation system uses an implicit-explicit procedure for the integration of dynamic equilibrium equations of motion in which the continuum elements, joint elements uses explicit and implicit time marching schemes respectively. The time step is variable and same for both implicit and explicit schemes based on explicit continuum elements stability criteria. Artificial viscosity has been used in the computation to achieve numerical stability.

Many impact problems have been modelled in low, medium and high velocity ranges. A detailed discussion on these results has been presented. Some blunt/smoothened projectile impact problems in high velocity impact have been simulated and the results are discussed. Parametric studies of blunt projectiles in high velocity impact are also presented.

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