

**A CRITICAL STUDY OF STRUCTURAL TOPOLOGY AND  
SHAPE OPTIMIZATION PROBLEMS USING SOFT  
COMPUTING TECHNIQUES AND ARTIFICIAL DENSITY  
METHOD**

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**INDIAN INSTITUTE OF TECHNOLOGY DELHI**

**MAY, 2009**

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METHOD**

by

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submitted  
in fulfillment of the requirements  
for the degree of  
**DOCTOR OF PHILOSOPHY**  
To



**INDIAN INSTITUTE OF TECHNOLOGY DELHI**  
**MAY, 2009**

**DEDICATED TO MY BELOVED PARENTS**

**FATHER SHRI. SHIV DARSHAN SINGH**

**MOTHER SMT. NISHA SINGH**

## **CERTIFICATE**

This is to certify that the thesis titled **A critical study of structural Topology and Shape Optimization problems Using Soft Computing Techniques and Artificial Density method** being submitted by Mr. Nidur Singh to the Indian Institute of Technology, Delhi for the award of Doctor of Philosophy in Applied Mechanics Department is a record of bonafide research work carried out by him. He has worked under my guidance and has fulfilled the requirements for the submission of thesis, which, in my opinion, has reached the requisite standard.

The results contained in this thesis have not been submitted in part or full, to any University or Institute for the award of degree or diploma.

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(Nidur Singh)

## **ABSTRACT**

Genetic Algorithm has been actively researched by our team at IIT delhi for its implementation for topology optimization from last many years and other researchers around the globe. Several strategies have been suggested for reduction of computational effort and these algorithms have been implemented which has lead to tremendous improvement of computational efficiency of genetic algorithm. Several new strategies have been proposed in this work to improve upon the computational efficiency of GA. Plates and Shells are the most widely used structures in civil, aerospace, marine and automobile industry as it uses the material in an efficient manner to resist external load. However, loading can be complex and they can occur in a variety of combination and it is essential to ensure that the structure is strong, stiff and resistant to failure while material economy is important.

In the present investigation GA has been used for topology optimization of plates and shell problems. The formulation considered is of compliance minimization subject to volume constraint and weight minimization subject to multiple loading conditions and several displacement constraints. The investigation is mainly focused on latter type formulation because it is more realistic and practical and analogous to actual problem situations. A critical comparison of method has been done with artificial density method. Formulation of weight minimization problem with several displacement constraint has been done in straight forward manner instead of weighted approach. The artificial density method has been implemented with sequential linear programming with move

limits solver. Topology optimization of plate and shells undergoing large deflection has been done with artificial density method. The derivative has been calculated using adjoint method and these derivatives are compared using finite difference method. Artificial neural network(ANN) has been investigated for implementation in genetic algorithm. Optimization frame work has been proposed for the implementation of ANN with GA.

## LIST OF CONTENTS

	Page
Certificate	i
Acknowledgements	ii
Abstract	iv
Table of Contents	vi
List of Figures	xv
List of Tables	xxvi
List of Symbols	xxviii

### CHAPTER 1: INTRODUCTION AND LITERATURE REVIEW

1.1. Introduction	1
1.2. Design Optimization Problem	3
1.3. Mathematical Programming Method	
1.3.1. Zoutendijk Feasible Direction Method	4
1.3.2. Gradient Projection Method	4
1.3.3. Generalized Reduced Gradient Methods (GRG)	4
1.3.4. Sequential Linear Programming	5
1.3.5. Sequential Quadratic Programming	5
1.3.6. Convex Linearization Method	6
1.3.7. Penalty Function Methods	6
1.3.8. Optimality Criteria Methods (OC)	7
1.4. Literature on Topology Optimization of Continuum Structures	7
1.4.1 Homogenization-based Approach	8
1.4.2. Solid Isotropic Microstructures with Penalization (SIMP) Method	13
1.4.3. Evolutionary Structural Optimization (ESO) (e-method)	14
1.4.4. Perimeter Constraint Based Approach	14
1.4.5. Hybrid Methods (h/e- methods)	15

1.4.6. Methods based on Shape Derivatives	16
1.5. Literature on Topology Optimization using Genetic Algorithm (GA)	19
1.5.1. Two Dimensional (2-D) Crossover operators	20
1.5.2. Voronoi Cell/Bar Representation Methods	20
1.5.3. Connected Domain Approaches	21
1.5.4. Issue of Structural Design connectivity in bit-array representation method	22
1.5.5. Design Domain Reduction	22
1.6 Literature on Topology Optimization using Genetic Algorithm (GA) and function approximation techniques(ANN)	22
1.7. Object and Scope of Present Work	25
1.8. Thesis Organization	27
 <b>CHAPTER 2: THREE PHASE HYBRID GENETIC ALGORITHM FOR 2D PROBLEM</b>	
2.1 Introduction	29
2.2 Mathematical Formulation of the Problem	31
2.2.1. Topology optimization problem	31
2.2.2. Ill posed nature of the topology optimization problem and remedy	33
2.2.3 Computation of perimeter	35
2.3. Bitmap representation and Genetic Algorithm	36
2.3.1. Block Crossover	37
2.3.2. Mutation	38
2.3.3. Selection Scheme	38
2.3.4. Elitism	39
2.4. Literature Review on Two phase adaptive GA for Topology Optimization	39
2.5 Two Stage Adaptive GA	42
2.6 Skeletonization	43
2.7 Three Phase Approach for Topology Optimization	44
2.7.1 Phase 1 (Bit array representation)	44

2.7.2. Phase 2 (GA based optimization using geometric variables for a generalized topology )	45
2.7.2.1 Geometric design variables for Second Phase	45
2.7.2.2 Selection of General Topology variables for the Second Phase	47
2.7.2.3 Setting of upper and lower limit for Design variables	50
2.7.3. Third Phase	51
2.7.3.1 Lower and Upper Limit Extraction	53
2.8 Numerical Results	54
2.8.1 Minimum Work Design Problems	55
2.8.2 Minimum Weight Design Problems	67
2.9 Comparison of Three Phase GA and Two Phase GA	79
2.10 Conclusions	79

### CHAPTER 3: ARTIFICIAL DENSITY METHOD FOR 2D TOPOLOGY OPTIMIZATION PROBLEMS FOR MINIMIZATION OF WEIGHT SUBJECT TO DISPLACEMENT CONSTRAINTS

3.1 Introduction	80
3.2 Detail description of SIMP method	81
3.2.1 Design variables	81
3.2.2 Solution Methods	83
3.2.2.1 Optimality Criteria	83
3.2.2.2 Sequential linear Programming Method with move limits	85
3.2.2.2.1 Basic Method	86
3.2.3 Complications	86
3.2.3.1 Mesh refinement and existence of solutions	87
3.2.3.2 Filtering the sensitivities	88
3.3 Mathematical Formulation of the Problem of Topology optimization problem in SIMP method	89
3.4 Methodology Description of Two Step SIMP method	90
3.5 Numerical results	93

3.5.1 Minimum Work Design Problems	93
3.5.2 Minimum Weight Design Problems	95
3.6 Comparison of SIMP and GA method	101
3.7 Conclusions	103

## CHAPTER 4: THREE PHASE HYBRID GENETIC ALGORITHM FOR PLATES AND SHELLS PROBLEM

4.1 Introduction	104
4.2 Literature Review of Topology Optimization of Plates and shells using GA.	104
4.3 Formulation of Shell Element	105
4.3.1 Degenerate Isoparametric Shell elements	105
4.3.2 Coordinate Systems	107
4.3.2.1 Global coordinate Set $x_i$	107
4.3.2.2 Nodal Unit Vectors	107
4.3.2.2 Curvilinear Coordinates	110
4.3.2.3 Local Coordinates $x_i'$	110
4.3.2.4 Element Geometry	111
4.3.2.5 Numerical Integration	113
4.3.2.5 Heterosis Element	114
4.3.2.6 Definitions of Strain	114
4.3.2.7 Strain Displacement matrix	115
4.3.2.7.1 Linear Strain displacement matrix $B_0$	115
4.3.2.8 Material property matrix or elasticity matrix D	119
4.3.2.9 Element Stiffness matrix	120
4.4 Numerical Results	121
4.4.1 Minimum Weight Design Problems	121
4.4.1.1 Clamped plate subjected to concentrated load at the centre and four at the centre of the each quadrant.	122
4.4.1.2 Simply supported plate subjected to concentrated load at the centre and four at the center of the each quadrant.	128

4.4.1.3 Minimization of weight of Spherical shell is subjected to concentrated load at the centre and four at the quarter points of the lines of symmetry	133
4.4.1.4 Minimization of weight of Spherical shell is subjected to concentrated load at the centre and four at the quarter points of the lines of symmetry	138
4.4.1.5 Minimization of weight of Cylindrical shell is subjected to concentrated load at the centre and four at the quarter points of the lines of symmetry.	142
4.4.2 Minimum Work Design Problems	148
4.4.2.1 Clamped plate subjected to concentrated load at the centre	148
4.4.2.2 Simply supported plate subjected to concentrated load at the centre	149
4.4.2.3 Minimization of Compliance of Spherical shell is subjected to concentrated load at the centre.	151
4.4.2.4 Minimization of Compliance of Cylindrical shell is subjected to concentrated load at the centre.	152
4.5 Conclusions	154

**CHAPTER 5: TWO STEP SIMP METHOD FOR TOPOLOGY OPTIMIZATION OF LINEAR PLATES AND SHELLS PROBLEMS UNDER MULTIPLE LOAD CONDITIONS AND MULTIPLE CONSTRAINTS**

5.1 Introduction	155
5.2 Detail description of SIMP method	156
5.3 Mathematical Formulation of the Problem of Topology optimization problem in SIMP method	156
5.4 Methodology Description of Two Phase SIMP method	156
5.5 Design Derivative Calculation using adjoint method.	157
5.5.1 One Design variable in one FE Element	158
5.5.2 Psuedocode for design derivative calculation	159



6.3 Geometric Representation Phase	196
6.3.1. Topology Description and Design Variables	196
6.3.2. Mathematical Formulation	198
6.4 Numerical Examples	199
6.5 Conclusions	205

## CHAPTER 7: TOPOLOGY OPTIMIZATION OF GEOMETRICALLY NONLINEAR SHELL PROBLEMS USING TWO STEP SIMP METHOD

7.1 Introduction	206
7.2 Finite Element formulation of geometrically Nonlinear Shell problem	
7.2.1 The Basic Problem	208
7.2.2 Linearized Incremental equation	209
7.2.3 Finite Element formulation of $K_T$ and $K_G$ matrix.	210
7.2.4 Psuedocode of Non-linear Analysis Code	213
7.2.5 Validation of Code	214
7.3 Problem Formulation	215
7.4 Adjoint Method for Sensitivity Analysis	215
7.4.1 Minimization of end-compliance	215
7.4.2 Validation of Design derivatives	217
7.4.2.1 One Variable in One FE Element	217
7.4.2.1 Four Variable in One FE Element	220
7.5 Numerical Results	
7.5.1 Compliance Minimization Problems	223
7.5.2 Weight Minimization Problems	
7.5.2.1 Minimization of weight of Spherical shell is subjected to concentrated load at the centre and three displacement constraints	225
7.5.2.2 Minimization of weight of Spherical shell is subjected to concentrated load at the centre and three different displacement constraints	227
7.5.2.3 Minimization of weight of Spherical shell is subjected to three loads and three different displacement constraints.	229

7.5.2.4 Minimization of weight of Spherical shell is subjected to three loads and three different displacement constraints	231
7.5 Conclusions	232
<b>CHAPTER 8: TOPOLOGY OPTIMIZATION USING GA, ARTIFICIAL NEURAL NETWORK AND WITH FINITE ELEMENT METHOD</b>	
8.1 Introduction	234
8.2 Artificial Neural Network (ANN)	235
8.2.1 The Generalized delta rule (Backpropagation learning)	237
8.3 Representation of string into input data for ANN training and Limitations	243
8.4 Features extractions of the designs	244
8.4.1 Computation of Feature Parameters	245
8.4.1.1 Area	245
8.4.1.2 Perimeter	246
8.4.1.3 Euler Number	248
8.4.1.4 Spatial moments	250
8.5 Optimization framework for topology optimization	250
8.6 Numerical Results	253
8.6.1 Problem 1: The 1x1 cantilever plate (assuming symmetry) One hole Design	254
8.6.2 Problem 2: The 1x1 cantilever plate (assuming symmetry) Four hole Design	256
8.7 Conclusions	258
<b>CHAPTER 9: CONCLUSIONS AND FUTURE SCOPE OF WORK</b>	
9.1. Summary and Conclusions	259
9.1.1 Three phase GA	259
9.1.2 Two Step SIMP method	262
9.1.3 Two phase method for 3D problem	263

9.1.4 Topology Optimization using Artificial Neural Network(ANN) and Genetic Algorithm(GA)	264
9.2 Future Scope of work	264
List of Reference	266
Paper Published	294
Biodata	295