STRUCTURED MAPPINGS AND PROCRUSTES PROBLEMS WITH THEIR APPLICATIONS IN CONTROL SYSTEMS

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STRUCTURED MAPPINGS AND PROCRUSTES PROBLEMS WITH THEIR APPLICATIONS IN CONTROL SYSTEMS

by

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Dedicated to

My Family

Certificate

This is to certify that the thesis entitled **"Structured Mappings and Procrustes Problems with their Applications in Control Systems"** submitted by **"Mr. Mohit Kumar Baghel"** to the Indian Institute of Technology Delhi, for the award of the Degree of **Doctor of Philosophy**, is a record of the original bonafide research work carried out by him under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted in part or full to any other university or institute for award of any degree or diploma.

New Delhi
June 2023

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Abstract

Structured mapping problems have occurred in many engineering applications due to their close relationship with the nearness problems involving matrices. These problems have been studied extensively in the literature. When the mapping problem is inconsistent, the Procrustes problem (least square solution to the problem) has also been studied over the years. These problems have been studied for various structures arising in the stability analysis for port-Hamiltonian systems.

An explicit solution (necessary and sufficient condition, characterization, and mappings with minimal Frobenius norm) to the dissipative mapping problem is derived. The minimal-norm dissipative mapping is then used to determine the distance to asymptotic instability for dissipative-Hamiltonian systems under general structure-preserving perturbations. Motivated by the perturbation analysis of port-Hamiltonian systems, a doubly structured mapping problem has been defined for various structure classes that include (skew)-Hermitian, (skew)-symmetric, pseudo(skew)-symmetric, pseudo(skew)-Hermitian, positive (semi)definite, and dissipative matrices. These mappings are then used to estimate eigenpair backward errors of matrix pencils with block and symmetry structure-preserving perturbations. These pencils arise in the passivity analysis of port-Hamiltonian systems. If the dissipative mapping is inconsistent, then the dissipative Procrustes problem (aka non-symmetric positive semidefinite Procrustes problem) has been studied for the matrix equation $AX = B$, $A, B \in \mathbb{R}^{n,m}$ in variable X. A semi-analytical/algorithm has been proposed to solve this problem with the desired accuracy. This algorithm also applies to solving the complex non-Hermitian positive semidefinite Procrustes problem, where $X, B \in \mathbb{C}^{n,m}$. The efficiency of the proposed algorithm is illustrated in several numerical examples. A modified non-symmetric positive semidefinite Procrustes problem has also been studied for matrix equations $AX = B$, $XC = D$ in variable X, where $A, B \in \mathbb{R}^{m,n}$ and $C, D \in \mathbb{R}^{n,m}$.

सार

मैट्रिसेस से जुड़ी निकटता की समस्याओं के साथ उनके घनिष्ठ संबंध के कारण कई इंजीनियरिंग अनुप्रयोगों में संरचित मानचित्रण समस्याएं उत्पन्न हुई हैं। साहित्य में इन समस्याओं का विस्तृत अध्ययन किया गया है। जब मानचित्रण की समस्या असंगत होती है, तो वर्षों से प्रोक्रस्ट्स समस्या (समस्या का कम से कम वर्ग समाधान) का भी अध्ययन किया गया है। पोर्ट-हैमिल्टनियन सिस्टम के लिए स्थिरता विश्लेषण में उत्पन्न होने वाली विभिन्न संरचनाओं के लिए इन समस्याओं का अध्ययन किया गया है।

विघटनकारी मानचित्रण समस्या के लिए एक स्पष्ट समाधान (आवश्यक और पर्याप्त स्थिति, लक्षण वर्णन, और न्यूनतम फ्रोबेनियस मानदंड के साथ मानचित्रण) प्राप्त किया गया है। सामान्य संरचना-संरक्षण क्षोभ के तहत विघटनकारी-हैमिल्टनियन प्रणालियों के लिए स्पर्शोन्मुख अस्थिरता की दूरी निर्धारित करने के लिए न्यूनतम-मानक अपव्यय मानचित्रण का उपयोग किया जाता है। पोर्ट-हैमिल्टनियन प्रणालियों के गड़बड़ी विश्लेषण से प्रेरित होकर, विभिन्न संरचना वर्गों के लिए एक दोगुनी संरचित मानचित्रण समस्या को परिभाषित किया गया है जिसमें (तिरछा)-हर्मिटियन, (तिरछा)-सममित, छद्म(तिरछा)-सममित, छद्म(तिरछा)-हर्मिटियन शामिल है। , सकारात्मक (अर्ध) निश्चित, और विघटनकारी मैट्रिक्स। इन मानचित्रणों का उपयोग तब ब्लॉक और समरूपता संरचना के साथ मैट्रिक्स पेंसिल के ईजेनपेयर बैकवर्ड एरर का अनुमान लगाने के लिए किया जाता है- गड़बड़ी को संरक्षित करना। ये पेंसिल पोर्ट-हैमिल्टनियन सिस्टम के निष्क्रियता विश्लेषण में उत्पन्न होती हैं। यदि विघटनकारी मानचित्रण असंगत है, तो विघटनकारी प्रोक्रस्ट्स समस्या (उर्फ गैर-सममित सकारात्मक अर्ध-निश्चित प्रोक्रस्ट्स समस्या) का अध्ययन मैट्रिक्स समीकरण $AX = B, A,B \in R$ चर X के लिए किया गया है। एक अर्ध-विश्लेषणात्मक / एल्गोरिथम को वांछित सटीकता के साथ इस समस्या को हल करने का प्रस्ताव दिया गया है। यह एल्गोरिदम भी

विसार जटिल गैर-हर्मिटियन सकारात्मक अर्ध-निश्चित प्रोक्रस्ट्स समस्या को हल करने के लिए लागू होता है, जहां $X, B \in C^{n,m}$। प्रस्तावित एल्गोरिथ्म की दक्षता कई संख्यात्मक उदाहरणों में सचित्र है। मैट्रिक्स समीकरणों $A X= B, X C = D$ के चर X में एक संशोधित गैर-सममित सकारात्मक अर्ध-निश्चित समस्या का भी अध्ययन किया गया है, जहां $A, B \in R^{m,n}$ और $C, D \in R^{n,m}$।

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