STABILITY OF BOUNDARY LAYER FLOW OVER A FLAT PLATE

by

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Department of Applied Mechanics
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C_E_R_T_I_F_I_C_A_T_E

This is to certify that the thesis entitled "STABILITY OF BOUNDARY LAYER FLOW OVER A FLAT PLATE" that is being submitted by Mr. T.K. Vashist for the award of the degree of Doctor of Philosophy, is a record of bonafide work carried out by him under my guidance and supervision. The material embodied in this thesis, unless acknowledged otherwise, has not been submitted in part or in full for any other Diploma or Degree of any University.

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(T.K. Vashist)

ABSTRACT

In this work the stability of boundary layer flow over a flat plate has been investigated. In particular the following two problems have been studied independently; (a) Non-Linear Stability based on the formalism of Stuart (1960) and Watson (1960); and (b) Linear Stability with inclusion of non-parallel effects due to growth of boundary layer.

For the non-linear problem a detailed investigation over a wide range in the (F-R) plane has been carried out. This is to identify the regions where subcritical or supercritical equilibrium can exist on account of the weakly non-linear effects of the type considered in the Stuart-Watson formalism. A large number of higher-order Landau coefficients of the Stuart-Landau series have been computed for the points in the (α-R) plane and these have been used to calculate equilibrium amplitude by using accelerated convergence techniques of Shanks (1955) and Padé Approximants (c.f. Van Dyke 1974). The modified neutral-curve on account of the weakly non-linear effects has been presented and it shows that while there is very little lowering of the critical Reynolds number, there is a substantial upward shift of the tip of the linear-theory neutral curve.

For the non-parallel problem, a completely new formulation has been developed based on the stipulation of "localised

similarity". The new formulation leads to a modified Orr-Sommerfeld equation, and unlike most of the past work, this formulation can be readily extended to non-linear work. The modified neutral curve on account of non-parallel effects shows both, a substantial decrease in critical Reynolds number as well as a considerable upward shift of the upper tip of the linear-theory neutral curve.

The work is expected to serve as a step towards reconciling some of the differences existing between the theoretical and experimental results on the stability of boundary layer flow over a flat plate. As future work, it is believed that it would be fruitful to study the combined non-parallel cum non-linear problem.

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