

# FINITE ELEMENT ANALYSIS OF NOSE CONE SHELLS FOR AEROELASTIC FLUTTER

by

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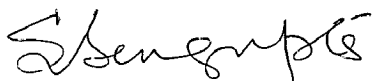
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Certificate

This is to certify that the thesis entitled, 'Finite Element Analysis of Nose Cone Shells for Aeroelastic Flutter' being submitted by Packiamani Joshua Sunder to the Indian Institute of Technology, Delhi, for the award of the Degree of Doctor of Philosophy in Applied Mechanics Department, is a record of bonafied research work carried out by him. He has worked under our guidance and has fulfilled the requirements for the submission of this thesis, which has reached the requisite standard.

The results contained in this thesis have not been submitted in part or full, to any other University or Institute for the award of any degree or diploma.

  
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ABSTRACT

This investigation is concerned with the analysis of nose cone shells for flutter using the finite element method. A survey is made of the several algorithms available in the literature for extracting the eigenvalues of unsymmetric matrices arising in linear panel flutter problems. The Block Stodola iteration technique and the static condensation procedure are found to be suitable. The two algorithms are compared from the point of view of computing time, economy and storage requirements.

Study has also been conducted by varying the parameters of the cone. Empirical formula for determining the optimum cone angle in resisting flutter has been presented. The resistance given to flutter by ogive shells have been compared with those of cones of equal initial and final radii and length. An empirical relationship for determining the flutter boundary of secant ogive shells is given in terms of tangent ogives and conical shells having same initial and final radii and length.

The effect of temperature on the flutter boundary has been studied alongwith internal pressure. It has been found that the flutter boundary considerably drops with increase in temperature and it is concluded that one of the reasons for the discrepancy in the tally between the experimental and theoretical values of flutter boundary could be the effect of temperature due to aerodynamic heating.

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