

**STATIC AND DYNAMIC ANALYSIS OF FUNCTIONALLY
GRADED PLATES AND SHELLS UNDER AERO-THERMO-
MECHANICAL LOADS**

by

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CERTIFICATE

This is to certify that the thesis entitled ***Static and Dynamic Analysis of Functionally Graded Plates and Shells under Aero-Thermo-Mechanical Loads***, which is being submitted by **Mr. T. Prakash** to the Indian Institute of Technology Delhi in fulfillment of the requirements for the award of the degree of ***Doctor of Philosophy*** is a bonafide record of original research work carried out by him under our guidance. The contents of this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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ABSTRACT

The present thesis addresses some of the problems dealing with the static and dynamic analyses of functionally graded plates and shells subjected to aero-thermo-mechanical loads using field-consistent and high precision plate/shell finite elements. The formulation is based on the first-order shear deformation theory and developed with reference to both the neutral and the mid- surfaces. The material properties vary through the thickness according to a power-law distribution and the locally effective material properties are evaluated using the Voigt rule of mixture or the Mori-Tanaka homogenization method. The geometric nonlinearity is incorporated using von Kármán's theory. The solutions are obtained using the Newton-Raphson iterative method/eigenvalue approach for the static problems and using the direct time integration procedure/eigenvalue formulation for the dynamic cases. The methodologies employed here are assessed, wherever possible, by studying the problems for which the solutions are available.

Firstly, as the position of neutral surface for FGM plates/shells varies depending on the material gradation, the effect of in-plane fixity and the approach of neutral/mid- surface based formulation are clearly brought out by investigating the bending behavior of FGM plates. Then, the post-buckling behavior of FGM plates and the appropriate solution procedures to obtain its characteristics depending on the boundary and loading conditions are discussed. The occurrence of secondary instability in the primary post-buckling path of FGM plate at higher load level is also established. Later, the nonlinear free and flutter vibration of FGM plates are examined under aero-thermal loads. The onset of change in equilibrium position, where a sudden drop occurs in the frequency value at higher amplitude of vibration, is also predicted. The flexural vibration of plates changing from limit cycle oscillation to chaotic motion with the increase in aerodynamic load is also examined using a time-response analysis. Here, the aerodynamic force is evaluated based on the third-

order piston theory. Finally, the nonlinear axi-symmetric dynamic buckling of shallow spherical caps is analyzed using an axi-symmetric shell element with a semi-analytical approach. Different loading profiles in the form of step load or triangular pulse of various durations are considered. In addition to the above nonlinear studies, the linear asymmetric vibration and buckling of FGM circular plates are also investigated as a special case by deducing the formulation of spherical shells. In all these problems, the parametric studies are performed considering various geometric/material parameters, and different types of loading and boundary conditions.

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