A THESIS ON

SOME PROBLEMS OF DISCONTINUITIES IN PHYSICAL GASDYNAMICS

By

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DEDICATED TO MY PARENTS

CERTIFICATE

This is to certify that the thesis entitled "Some Problems of Discontinuities in Physical Gas Dynamics" which is being submitted by Mr. Sunil Kumar Jain for the award of degree of Doctor of Philosophy in Mathematics to the Indian Institute of Technology, Delhi, is a record of bonafide research work carried out by him under my guidance and supervision.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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SYNOPSIS

This thesis entitled "Some Problems of Discontinuities in Physical Gas Dynamics" is divided into six chapters. Chapter I presents an introduction to the thesis. Chapter II deals with the propagation of weak discontinuities in thermally conducting and dissociating gases. Chapter III discusses the formation of shock waves in Physical Gasdynamics. Chapter IV deals with the problem of breakdown of acceleration waves in radiative and vibrational non-equilibrium gasdynamics. Chapter V devotes to study some aspects of shock waves in physical gasdynamics. The last one namely Chapter VI is concerned with the geometry of three dimensional radiative gas flows. A brief description of each chapters is given below:

Chapter I: Introduction

In this chapter, a brief account of the fundamental ideas and features characterizing the gases coming under our review is given. This is followed by a brief historical survey of the previous studies made related to the present work.

Chapter II: Propagation of Weak Discontinuities in Thermally Conducting* and Dissociating Gases**

Using singular surface theory, the phenomena associated with the uniform as well as non-uniform propagation of weak

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discontinuities through thermally conducting and dissociating gases are studied. The basic differential equations governing the growth and decay of weak discontinuities are derived and solved completely. The criterion for decay or blow up of these discontinuities is obtained. It turns out that the thermal conduction and dissociation allow the existence of a singular surface carrying a weak discontinuity which grows into a shock and the role of dissociation and thermal conduction is to cause rapid damping in the formation of this shock.

Chapter III: Formation of Shock Waves in Physical Gas Dynamics

Following the analysis as given in Chapter II, we have studied in this chapter the formation of shock waves i.e. the termination of weak discontinuities into a shock wave in Physical Gasdynamics. We have divided the work into sections I and II respectively. In Section 1, we have studied the entitled problem in vibrational and radiative non-equilibrium gasdynamics whereas in Section II, we have carried out the same analysis to radiation magnetogasdynamics. The differential equations for growth and decay of weak discontinuities have been formulated here. This class of equations have been solved completely and particular cases of plane and spherical waves have been discussed. The special and temporal extents of these discontinuities to generate into a plane and spherical shock waves have been indicated. It turns out that the vibrational and radiative non-equilibrium allows the existence of a

singular surface carrying a weak discontinuity into such uniform medium. This weak discontinuity may grow into a shock and the role of vibrational non-equilibrium is to decrease the critical time for the formation of this shock whereas the effect of radiation heat transfer energy is to contribute damping in the formation of this shock wave.

Chapter IV: On the Breakdown of Acceleration Waves in Vibrational and Radiative Non-Equilibrium Gasdynamics

This chapter is devoted to the study of characteristic solution in the neighbourhood of the leading frozen characteristics in relaxing and radiating gas flows. It is found that at the cusp of the envelope of intersecting forward characteristics, there occurs breakdown of the wave after a finite critical time. It is observed that there exists a critical value of the initial amplitude of the wave such that all compressive waves with an initial amplitude greater than the critical one will terminate into a shock wave due to non-linear steepening while an initial amplitude less than the critical one will result in a continuous decay.

Chapter V: On Some Aspects of Shock Waves in Physical Gasdynamics

This chapter contains two sections. In the first section, based on Truesdell's analysis, the differential effect

of shock fronts in non-equilibrium gasdynamics has been studied. While in the second section, the same analysis has been extended to study the shock waves in magnetohydrodynamic flow with heat addition. The differential equations governing the flow have been integrated to represent the jump conditions across shock wave. These jump conditions have been solved for the density strength of the shock wave. As a result of this, the differential effects of shock front have been discussed.

Chapter VI: On the Geometry of Radiative Gas Flows

In this chapter, we have discussed the geometry of stream lines in radiation gasdynamics. The purpose of this chapter is to investigate various dynamical and kinematical relations connecting the flow variables with the geometrical parameters of the stream line trajectories. The expressions for the tangent, principal normal and binormals vectors and the curvature and torsion of the stream line are given in terms of the velocity components, the pressure, the density and the radiation. The variation of pressure along stream lines, their principal normals and binormals are obtained. It turns out that the radiating character of the gas decreases the pressure gradient along the stream line. However, the pressure remains constant along the binormals. If the stream lines are straight lines, the trajectories of the principal normals lie in the surface of the constant pressure. Finally, the expressions for vorticity components are found in terms of curvature of the stream lines, their principal normals and binormals.

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