

**EFFICIENT MULTIFIELD FINITE ELEMENT
FORMULATION WITH LAYERWISE MECHANICS FOR
SMART PIEZOLAMINATED SHELLS FEATURING
DELAMINATIONS AND DEBONDING**

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**DEPARTMENT OF APPLIED MECHANICS
INDIAN INSTITUTE OF TECHNOLOGY DELHI
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SMART PIEZOLAMINATED SHELLS FEATURING
DELAMINATIONS AND DEBONDING**

by

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Department of Applied Mechanics

Submitted

in fulfilment of the requirements of the degree of Doctor of Philosophy

to the



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*Dedicated to
my family*

Certificate

This is to certify that the thesis entitled “**Efficient Multifield Finite Element Formulation with Layerwise Mechanics for Smart Piezolaminated Shells Featuring Delaminations and Debonding**” being submitted by **Mr. Adnan Ahmed** to the Indian Institute of Technology Delhi, for the award of the degree of Doctor of Philosophy in Applied Mechanics is a record of original bonafide research work carried out by him under our supervision and guidance. The thesis work, in our opinion, has reached the requisite standard fulfilling the requirements for the degree of Doctor of Philosophy.

The results contained in this thesis have not been submitted in part or full to any other University or Institute for the award of any degree or diploma.

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Abstract

Anisotropic fiber-reinforced composite and soft-core sandwich laminates are finding increased usage in modern structures such as aircrafts, spaceships, automobiles, marine propellers, wind turbine blades and bridges. For their improved performance and reliability, it is often required to have inbuilt advanced features like active vibration control, adaptive shape control and real time structural health monitoring (SHM), by equipping them with surface-bonded or embedded piezoelectric patch sensors and actuators. Such smart laminated structures are, however, prone to delamination damages in the host laminate and debonding of transducers. Computationally efficient and accurate models for these structures are essential particularly for problems involving their dynamics, vibration control and wave propagation analysis, and for the inverse problem of damage detection in model-based SHM methods.

In this thesis, a new efficient multifield finite element (FE) formulation with layerwise mechanics is presented for the analysis of smart piezolaminated shells featuring multiple delaminations and transducer debondings. It represents the first two-dimensional (2D) model presented for this purpose. The laminate theory is based on the efficient coupled zigzag theory (ZIGT) considering a layerwise description of the inplane displacements and the normal deformation in the piezoelectric layers due to the transverse piezoelectric coefficient (d_{33} -effect), without introducing any additional displacement variables beyond the smeared third order theory. A piecewise quadratic through-thickness variation of the electric potential along with the provision of satisfying the equipotential condition of electroded surfaces is adopted. A quadrilateral facet shell element is developed having four kinetic nodes and one electric node which allows equipotential condition over a number of elements under an electrode resulting in a significant reduction in the number of electric degrees of freedom (DOFs). The local axes of the facet shell element are taken in such a way that they are optimally parallel to either set of its opposite sides. The delaminations and debondings are modeled using the region approach based on the free mode assumption.

A novel hybrid point-least squares (HPLS) method is proposed for satisfying the continuity of the nonlinear displacement field at the delamination, debonding and patch fronts. The continuity conditions are imposed exactly by relating the nodal DOFs of the adjacent elements at these fronts through a transformation matrix. Before imposing the continuity conditions, the nodal DOFs of the elements in the secondary segment are transformed to the local coordinate system of the respective adjacent elements in the intact/primary segment. The methodology is generic for multiple patch transducers, delaminations and debondings, occurring at any arbitrary interface and inplane locations. For active vibration control, the FE model is transformed to the reduced order modal space considering the first few modes, and is expressed in the state space format. The output feedback linear quadratic Gaussian (LQG) optimal controller is adopted for the vibration control.

The developed element is made to pass through the three-problem standard obstacle course test and is found to be among the best performers. By analyzing ultra thin shells, it is also established that the element is free of shear and membrane locking problems. Next, the element is critically assessed for its accuracy for the static and free vibration response of elastic singly and doubly curved composite and highly inhomogenous sandwich deep shells. The multiphysics element is also assessed for the stress analysis under mechanical and electric potential loadings, and for the free vibration response of various smart piezolaminated deep shells with different boundary conditions, covering both actuator and sensory roles of the piezoelectric layers. The results of all the problems considered are shown to be very accurate in comparison with the analytical 3D piezoelectricity solutions or full-field 3D FE solutions for both the elastic and piezolaminated smart shells. The performance of the element is then assessed for the free vibration analysis of beams, plates and shells with multiple delaminations in comparison with available theoretical and experimental results, and newly obtained 3D FE solutions. The relative performance of the proposed HPLS method and the other available methods of satisfying continuity is also studied. The accuracy of the element is then examined for the free vibration response of smart laminated plates and shells with delaminations and debondings of patch transducers. Finally, the effect of delaminations and transducer debondings on the active vibration control behavior of smart laminated plates and shells is studied. The results have shown that the developed coupled-field FE provides a simple, computationally efficient, robust and accurate tool for dynamics and control of moderately thick to thin multilayered smart plates and shells featuring delaminations and transducer debondings.

सारांश

असमदैशिक फाइबर प्रबलित समग्र और परतीय नरम-आन्तरक सैंडविच संरचना के उपयोग की एयरक्राफ्ट, स्पेसशिप, ऑटोमोबाइल, समुद्री प्रोपेलर, हवाई टरबाइन ब्लेड और पुल जैसे आधुनिक संरचनाओं में वृद्धि हुई है। इनके बेहतर प्रदर्शन और विश्वसनीयता के लिए, उन्हें सतह बंधुआ या एम्बेडेड पीजोइलेक्ट्रिक पैच सेंसर और एक्ट्यूएटर के साथ लैस करके, सक्रिय कंपन नियंत्रण, अनुकूली आकार नियंत्रण और वास्तविक, समय संरचनात्मक स्वास्थ्य निगरानी (एसएचएम) जैसी उन्नत सुविधाओं को अंतर्निहित करने की अक्सर आवश्यकता होती है। हालांकि, मेजबान परतीय और ट्रांसड्यूसर्स के डिबगिंग को नुकसान पहुंचाने के लिए, इस तरह की स्मार्ट लैमिनेटेड संरचनाएं प्रवण हैं। अभिकलित्रिय, उनकी गतिशीलता, कंपन नियंत्रण और तरंग प्रसार विश्लेषण, और उलटा समस्या के लिए मॉडल-आधारित एस एच एम विधियों में क्षति का पता लगाना, इन संरचनाओं के लिए कुशल और सटीक मॉडल विशेष रूप से शामिल समस्याओं के लिए आवश्यक हैं।

इस शोध निबंध में, परतदार यांत्रिकी के साथ एक नया कुशल बहुक्षेत्रीय परिमित तत्व सूत्रीकरण कई गैर-परतबंदी वाले स्मार्ट पिजोलेमिनेटेड गोले और ट्रांसड्यूसर असम्बन्धी के विश्लेषण के लिए प्रस्तुत किया गया है। इस काम के लिए, यह पहले दो-आयामी (2D) मॉडल का प्रतिनिधित्व करने के लिए प्रस्तुत किया गया है। परतीय सिद्धांत कुशल युग्मित सिद्धांत (ZIG) पर आधारित है, विमान विस्थापन और सामान्य विरूपण में पिजोइलेक्ट्रिक परत अनुप्रस्थ पिजोइलेक्ट्रिक गुणांक (डी 33-प्रभाव) के कारण, बिना परिचय के स्मियर किए गए तीसरे ऑर्डर सिद्धांत से परे कोई अतिरिक्त फैलाव चर में एक परतवार विवरण पर विचार किया गया है। एक टुकड़ा संतोषजनक के प्रावधान के साथ-साथ विद्युत क्षमता का द्विघात थोट-वैरिएशन इलेक्ट्रोडेड सतहों की लैस स्थिति को अपनाया जाता है। एक चतुर्भुज मुख का खोल तत्व को चार गतिज नोड्स और एक इलेक्ट्रिक नोड के रूप में विकसित किया गया है जो कि संचार को अनुमति देता है एक इलेक्ट्रोड के तहत तत्वों की एक संख्या पर हालत में एक महत्वपूर्ण कमी में जिसके परिणामस्वरूप स्वतंत्रता की विद्युत डिग्री की संख्या (डीओएफ)। पहलू खोल तत्व के स्थानीय अक्षों को लिया जाता है इस तरह से कि वे इसके विपरीत पक्षों के किसी भी सेट के लिए समानांतर रूप से समानांतर हैं। परिसीमन और फ्रीजिंग मोड के आधार पर रीजनिंग एप्रोच का उपयोग करके मॉडलिंग की जाती है। परिशोधन, डिबस्टिंग और पैच मोर्चों पर अरेखीय विस्थापन क्षेत्र की निरंतरता को संतुष्ट करने के लिए एक उपन्यास हाइब्रिड बिंदु-न्यूनतम वर्ग (एचपीएलएस) विधि प्रस्तावित है। एक परिवर्तन मैट्रिक्स के माध्यम से इन मोर्चों में निरंतरता आस-पास के तत्वों के नोडल डीओएफ से संबंधित स्थितियों को ठीक से लगाया जाता है। निरंतरता की शर्तों को लागू करने से पहले, संबंधित / प्राथमिक खंड में संबंधित आसन्न तत्वों की प्रणाली के माध्यमिक खंड में तत्वों के नोडल डीओएफ

को स्थानीय समन्वय में बदल दिया जाता है। कार्यप्रणाली है कई पैच ट्रांसड्यूसर, परिसीमन और डिबेंडरिंग के लिए सामान्य, किसी भी मनमाने तरीके से इंटरफ़ेस और विमान स्थान। सक्रिय कंपनी नियंत्रण के लिए, FE मॉडल को रूपांतरित किया जाता है पहले कुछ मोड पर विचार करते हुए ऑर्डर मोडल स्पेस कम किया गया, और राज्य स्पेस में व्यक्त किया गया प्रारूप। कंपनी नियंत्रण, आउटपुट फीडबैक लीनियर क्वाड्रैटिक गाऊसी (LQG) इष्टतम कंट्रोलर के लिए अपनाया जाता है।

विकसित तत्व तीन-समस्या मानक बाधा कोर्स से गुजरने के लिए बना है और सबसे अच्छा प्रदर्शन करने वालों में पाया जाता है। बहुत पतले खोल का विश्लेषण करके, यह भी स्थापित है कि तत्व कतरनी और झिल्ली लॉकिंग समस्याओं से मुक्त है। अगला, तत्व का लोचदार एकल और स्थिर, मुक्त कंपनी प्रतिक्रिया के लिए और इसकी सटीकता के लिए दोगुनी घुमावदार समग्र और अत्यधिक अमानवीय सैंडविच गहरे खोल में गंभीर रूप से मूल्यांकन किया गया है। बहुबिधि तत्व यांत्रिक और बिजली के संभावित लोडिंग के तहत तनाव विश्लेषण के लिए भी मूल्यांकन किया जाता है, और विभिन्न सीमा के साथ विभिन्न स्मार्ट पिजोलेमिनेटेड गहरे खोल के मुक्त कंपनी प्रतिक्रिया के लिए स्थितियां, पीजोइलेक्ट्रिक परतों के दोनों उत्प्रेरक और संवेदी भूमिकाओं को कवर करती हैं। परिणाम माना जाता है कि इलास्टिक और पाइजोलिनेटेड दोनों के लिए 3 डी पाईज़ोएलास्टिकिटी सॉल्यूशंस या फुल-फील्ड 3 डी एफई समाधान स्मार्ट खोल का सभी समस्याओं को विश्लेषणात्मक की तुलना में बहुत सटीक दिखाया गया है। तत्व के मुक्त कंपनी विश्लेषण के लिए तत्व के प्रदर्शन का आकलन उपलब्ध सैद्धांतिक की तुलना में कई परिसीमन वाले बीम, प्लेट और गोले और प्रयोगात्मक परिणाम, और नव प्राप्त 3 डी एफई समाधान करके किया जाता है। सापेक्ष प्रदर्शन प्रस्तावित एचपीएलएस विधि और निरंतरता को संतुष्ट करने के अन्य उपलब्ध तरीकों का भी अध्ययन किया गया है। तत्व की सटीकता को प्लेट और शेल ट्रांसड्यूसर्स के परिसीमन और डिबेंडरिंग के साथ स्मार्ट लैमिनेटेड की मुक्त कंपनी प्रतिक्रिया के लिए जांच की जाती है। अंत में, प्रभाव स्मार्ट के सक्रिय कंपनी नियंत्रण व्यवहार पर परिसीमन और ट्रांसड्यूसर डिबांडिंग टुकड़े टुकड़े प्लेटों और गोले का अध्ययन किया जाता है। परिणामों से पता चला है कि विकसित युग्मित क्षेत्र FE गतिशीलता और नियंत्रण के लिए एक सरल, कम्प्यूटेशनल रूप से कुशल, मजबूत और सटीक उपकरण प्रदान करता है मध्यम से पतले बहुस्तरीय स्मार्ट प्लेटों और गोले के लिए मोटी होती है, जिसमें परिसीमन होता है।

Contents

Certificate	i
Acknowledgements	ii
Abstract	iii
Contents	vii
List of Figures	xiv
List of Tables	xix
1 Introduction	1
1.1 PREFACE	1
1.2 LITERATURE REVIEW	2
1.2.1 Elastic laminated shells	3
1.2.2 Piezolaminated smart shells	6
1.2.3 Modeling delaminations in elastic laminated plates and shells	10
1.2.4 Modeling delaminations in piezolaminated smart structures	15
1.2.5 Active vibration control of piezolaminated smart structures featuring de- lamination	19

1.3	MOTIVATION	21
1.4	OBJECTIVES	22
1.5	ORGANISATION OF THE THESIS	23
2	Efficient Layerwise FE Formulation for Doubly Curved Piezolaminated Smart Shells	28
2.1	INTRODUCTION	28
2.2	CONSTITUTIVE EQUATIONS FOR PIEZOELECTRIC LAMINA	29
2.3	LOCAL COORDINATE SYSTEM	31
2.4	COUPLED ELECTROMECHANICAL ZIGZAG THEORY	32
2.5	COUPLED-FIELD FE FORMULATION	36
2.5.1	Extended Hamilton's principle	36
2.5.2	Interpolation functions and element matrices	39
2.5.3	Transformation from local to global coordinate system	42
2.5.4	Global equation of motion	43
2.6	CLOSURE	45
3	Static and Free Vibration Response of Elastic and Piezolaminated Smart Shells	46
3.1	INTRODUCTION	46
3.2	ISOTROPIC SHELLS	47
3.2.1	The obstacle course	47
3.2.2	Pinched hyperbolic paraboloid	54

3.3	STATIC RESPONSE OF ELASTIC LAMINATED SHELLS	56
3.3.1	Composite cylindrical shell	56
3.3.2	Composite spherical shell	57
3.3.3	Sandwich cylindrical shell	59
3.3.4	Shell with non-simply supported boundary conditions	60
3.4	FREE VIBRATION RESPONSE OF ELASTIC LAMINATED SHELLS	61
3.4.1	Composite cylindrical and spherical shell	62
3.4.2	Sandwich cylindrical shell	62
3.4.3	Shell with non-simply supported boundary conditions	64
3.5	STATIC RESPONSE OF PIEZOLAMINATED SMART SHELLS	64
3.5.1	Validation	66
3.5.2	Hybrid composite cylindrical shell	67
3.5.3	Hybrid sandwich cylindrical shell	72
3.6	FREE VIBRATION RESPONSE OF PIEZOLAMINATED SMART SHELLS . .	75
3.6.1	Validation	75
3.6.2	Hybrid composite cylindrical shell	75
3.6.3	Hybrid sandwich cylindrical shell	76
3.6.4	Hybrid sandwich cylindrical shell with non-simply supported boundary conditions	77
3.7	CONCLUSIONS	78

4 A Novel Hybrid Continuity Method for Modeling Delamination in Composite

Plates based on TOT	81
4.1 INTRODUCTION	81
4.2 FE FORMULATION BASED ON THE TOT	81
4.3 CONTINUITY CONDITIONS AT THE DELAMINATION FRONT	84
4.3.1 Point-continuity method	85
4.3.2 Least squares method	86
4.3.3 Hybrid point-least squares method	87
4.4 NUMERICAL RESULTS	88
4.4.1 Free vibration of composite beams	88
4.4.2 Thick composite beams	90
4.4.3 Stress analysis of composite plates	91
4.4.4 Free vibration of square composite plates with cutout and delamination . .	93
4.4.5 Free vibration of composite skew plates with cutout and delamination . .	96
4.5 CONCLUSIONS	99
5 Efficient Layerwise FE Formulation for Piezolaminated Smart Shells Featur-	
ing Delaminations and Transducer Debonding	100
5.1 INTRODUCTION	100
5.2 FE MODELING BASED ON COUPLED-FIELD ZIGT	101
5.3 CONTINUITY CONDITIONS AT THE DELAMINATION	
/DEBONDING/PATCH FRONTS	105
5.3.1 Point-continuity method	105
5.3.2 Least squares method	110

5.3.3	Hybrid point-least squares method	112
5.4	GLOBAL EQUATION OF MOTION	114
5.5	ACTIVE VIBRATION CONTROL	115
5.5.1	State space representation	116
5.5.2	LQG optimal control	116
5.6	CLOSURE	118
6	Response of Elastic Composite and Sandwich Plates and Shells Featuring Multiple Delaminations	120
6.1	INTRODUCTION	120
6.2	LAMINATED PLATES WITH DELAMINATIONS	121
6.2.1	Stress and free vibration analysis of simply supported laminated plates without delamination	122
6.2.2	Free vibration of composite beams with multiple delamination	122
6.2.3	Free vibration of sandwich plates	124
6.2.4	Stress analysis of sandwich plates	128
6.2.5	Free vibration of sandwich plates with two delaminations	132
6.3	LAMINATED SHELLS WITH MULTIPLE DELAMINATIONS	133
6.3.1	Composite spherical shell having delamination around a central cutout . .	134
6.3.2	Cylindrical sandwich shell with multiple delaminations	135
6.4	CONCLUSIONS	143
7	Active Vibration Control of Smart Plates and Shells with Delaminations and Transducer Debonding	145

7.1 INTRODUCTION	145
7.2 FREE VIBRATION RESPONSE	146
7.2.1 Validation	146
7.2.2 Delaminated smart composite cylindrical shell with partially debonded patch actuators/sensors	149
7.2.3 Smart sandwich plate with patch actuators and sensors featuring multiple delaminations	153
7.3 ACTIVE VIBRATION CONTROL	157
7.3.1 Smart composite plate with delamination and multiple transducer debondings	157
7.3.2 Smart composite shell with delamination and multiple transducer debondings	170
7.4 CONCLUSIONS	176
8 Conclusions	182
8.1 CONCLUSIONS FROM THE PRESENT WORK	182
8.2 FUTURE SCOPE OF WORK	188
References	189
A	212
A.1 LINEAR 3D CONSTITUTIVE EQUATIONS	212
B	215
B.1 DERIVATION OF $\mathbf{R}^k(z)$ MATRIX FOR ZIGT	215
C	218

CONTENTS

C.1 IDKQ INTERPOLATION OF DEFLECTION	218
List of Publications from the Thesis	224
Brief Biodata of the Author	225

List of Figures

1.1	A beam with through width delamination (a) The delaminated beam divided into four regions (b) The FE model of the delaminated beam using two-node beam elements.	12
2.1	(a) Shell surface as an assembly of facet elements (b) Four-node facet shell element showing electric node and local coordinate axes	32
2.2	(a) A doubly curved piezolaminated shell (b) Geometry of a hybrid laminate . .	33
3.1	Pinched cylinder problem	48
3.2	Comparison of normalized deflection for the pinched cylinder problem.	49
3.3	Scordelis-Lo roof under self weight.	50
3.4	Comparison of normalized deflection for the Scordelis-Lo roof under self weight. .	51
3.5	Pinched hemisphere with 18° hole	52
3.6	Comparison of normalized deflection for the pinched hemisphere with 18° hole .	53
3.7	One-eighth of the pinched cylinder modeled using uniform mesh gradient	54
3.8	(a) Pinched hyperbolic paraboloid (b) a quarter modeled using uniformly refined mesh	55
3.9	A quarter cylindrical shell panel under potential load	67
3.10	Variation of the midsurface radial deflection of a composite PZT cylindrical shell panel along the circumference	67

LIST OF FIGURES

3.11	Lay-ups for various hybrid laminates considered	69
4.1	(a) Composite plate with multiple delaminations across thickness and along span. (b) Geometry of a sublaminates	83
4.2	(a) Elements at a delamination front (b) Inplane displacement variations at intact laminates and sublaminates at the delamination front	84
4.3	Delamination locations considered for the composite $(0^\circ/90^\circ)_{2s}$ beam	89
4.4	Simply supported composite plate with through-width delamination under point load	92
4.5	Through-thickness variations of inplane displacements and stresses across the lower sublaminates in a simply supported delaminated composite plate	94
4.6	Composite plates with central cutout and delamination. (a) Square plate, (b) Skew plate	96
4.7	Present FE solution for the mode shapes of a 45° skew composite plate with cutout and 70% delamination	97
4.8	3D FE solution for the mode shapes of a 45° skew composite plate with cutout and 70% delamination	98
5.1	(a) A doubly curved laminated shell with delamination and transducer debonding (b) Geometry of a sublaminates separated by delamination/debonding interfaces	102
5.2	(a) Elements at a delamination front (b) Inplane displacement variations at intact laminates and sublaminates at the delamination front	106
5.3	A smart piezolaminates with patch transducer featuring debonding	108
6.1	Locations of delaminations in a composite beam	123
6.2	Soft-core sandwich plate with a central delamination	125

6.3	Mode shapes of a moderately thick ($S = 10$) square sandwich plate with a central delamination	127
6.4	Double-core sandwich plate with through-width delamination	129
6.5	Longitudinal variations of deflection and inplane stresses at the top and bottom surfaces of a delaminated sandwich plate under uniformly distributed load	130
6.6	Through-thickness distribution of stresses at different locations for the upper delaminated sublaminate.	131
6.7	Soft-core sandwich plate with multiple delaminations	132
6.8	Composite spherical shell panel with a central cutout and delamination around it	135
6.9	Sandwich cylindrical shell with multiple delaminations	139
6.10	Present FE solution for mode shapes for sandwich cylindrical shell with multiple delaminations	141
6.11	3D FE solution for mode shapes for sandwich cylindrical shell with multiple delaminations	142
7.1	A cantilever aluminum plate with partially debonded actuators	148
7.2	Cantilever smart cylindrical shell featuring delamination and partial debonding of patch piezoelectric actuators/sensors	149
7.3	Present ZIGT FE solution for mode shapes of smart composite shell ($S = 50$) with delamination and partial debonding in actuators and sensors	151
7.4	3D FE solution for mode shapes of smart composite shell ($S = 50$) with delamination and partial debonding in actuators and sensors	152
7.5	A smart sandwich plate integrated with patch actuator at top and sensor at bottom, and featuring multiple delaminations: (a) Geometry of the smart plate, (b) Single delamination case, (c) Multiple delaminations-case 1, (d) Multiple delaminations-case 2	154

LIST OF FIGURES

7.6	Present ZIGT solution for mode shapes of smart sandwich plate bonded with patch actuator and sensor, featuring multiple delaminations-case 1 for $\bar{S} = 100$	155
7.7	3D FE solution for mode shapes of smart sandwich plate bonded with patch actuator and sensor, featuring multiple delaminations-case 1 for $\bar{S} = 100$	156
7.8	Cantilever smart plate featuring delamination and partial debonding of patch piezoelectric actuators/sensors	158
7.9	Present ZIGT FE solution for mode shapes of smart composite plate with delamination	159
7.10	Present ZIGT FE solution for mode shapes of smart composite plate with partial debonding in actuators	160
7.11	Closed-loop responses of smart cantilever composite plate with delamination under step excitation of 0.4 N	162
7.12	Closed-loop responses of smart cantilever composite plate with transducer debonding under step excitation of 0.4 N	163
7.13	Closed-loop responses of smart cantilever composite plate with delamination and transducer debonding under step excitation of 0.4 N	164
7.14	Closed-loop responses of smart cantilever composite plate with delamination under impulse excitation of 0.0075 N-s	166
7.15	Closed-loop responses of smart cantilever composite plate with transducer debonding under impulse excitation of 0.0075 N-s	167
7.16	Closed-loop responses of smart cantilever composite plate with delamination and transducer debonding under impulse excitation of 0.0075 N-s	168
7.17	Cantilever smart shell featuring delamination and partial debonding of patch piezoelectric actuators/sensors	171
7.18	Closed-loop responses of smart cantilever composite shell with delamination under step excitation of 30 N	172

7.19 Closed-loop responses of smart cantilever composite shell with transducer debonding under step excitation of 30 N	173
7.20 Closed-loop responses of smart cantilever composite shell with delamination and transducer debonding under step excitation of 30 N	174
7.21 Closed-loop responses of smart cantilever composite shell with delamination under impulse excitation of 0.075 N-s	177
7.22 Closed-loop responses of smart cantilever composite shell with transducer debonding under impulse excitation of 0.075 N-s	178
7.23 Closed-loop responses of smart cantilever composite shell with delamination and transducer debonding under impulse excitation of 0.075 N-s	179
A.1 Reference axes (x, y, z) and material principal axes (x_1, x_2, x_3) for an orthotropic material	213
C.1 Nine-node Lagrange element for interpolation of rotations	220

List of Tables

3.1	Pinched cylinder: Normalized deflection at point B	48
3.2	Scordelis-Lo roof under self weight: Normalized deflection at point D	51
3.3	Pinched Hemispherical shell with 18° hole: Normalized deflection at point B . . .	53
3.4	Normalized deflection for the three test problems with 28 elements per side . . .	54
3.5	Pinched hyperbolic paraboloid with uniformly refined mesh: Normalized deflection at point A	56
3.6	Material properties	57
3.7	Deflection and stresses of composite (0°/90°/0°) cylindrical shell panel (Ren's panel) under sinusoidally distributed load	58
3.8	Nondimensional central deflection of simply supported composite spherical shell .	59
3.9	Nondimensional central deflection and stresses of simply supported long sandwich cylindrical shell under sinusoidally distributed load	61
3.10	Nondimensional central deflection \bar{W} for composite cylindrical shells under uniformly distributed load with various boundary conditions	61
3.11	Nondimensional fundamental frequency $\bar{\omega}_1$ of simply supported composite cylindrical and spherical shells	63
3.12	Nondimensional frequencies $\bar{\omega}_n$ of simply supported sandwich cylindrical shells .	65

3.13 Nondimensional frequencies $\bar{\omega}_n$ of composite complete cylindrical shells with non-simply supported boundary conditions	66
3.14 Material properties	68
3.15 Deflection, stresses and sensory electric potential of simply supported hybrid composite cylindrical shell ($R/a = 1$) under sinusoidal pressure load	70
3.16 Deflection, stresses and electric displacement of simply supported hybrid composite cylindrical shell ($R/a = 1$) under uniform potential load	71
3.17 Deflection, stresses and electric potential of simply supported long hybrid sandwich cylindrical shell ($R/a = 1$) under sinusoidal pressure load	73
3.18 Deflection, stresses and electric displacement of simply supported long hybrid sandwich cylindrical shell ($R/a = 1$) under uniform potential load	74
3.19 Nondimensional fundamental frequency ($\tilde{\omega} = \omega_1 \sqrt{\rho/C_{11}}$) of simply supported laminated piezoelectric spherical shell ($R/a = 1$)	75
3.20 Nondimensional frequencies of simply supported hybrid composite cylindrical shell ($R/a = 1$)	76
3.21 Nondimensional frequencies of simply supported hybrid sandwich cylindrical shell ($R/a = 1$)	77
3.22 Nondimensional frequencies of hybrid sandwich cylindrical shell ($R/a = 1$) with non-simply supported boundary conditions	79
4.1 Material properties	89
4.2 Fundamental frequencies (Hz) of cross-ply $(0^\circ/90^\circ)_{2s}$ cantilever beams with delamination	90
4.3 Natural frequencies (Hz) of thick ($S = 5$) cross-ply $(0^\circ/90^\circ)_s$ beams with delamination	91
4.4 Nondimensional natural frequencies of square plates with a cutout and delamination	95

LIST OF TABLES

4.5	Natural frequencies (Hz) of skew plates with a cutout and delamination	98
6.1	Material properties	121
6.2	Deflection, stresses and fundamental frequency of a three-layer simply supported plate	122
6.3	Natural frequencies (Hz) of cantilever cross-ply $(0^\circ/90^\circ)_{2s}$ beams with multiple delaminations	124
6.4	Nondimensional frequencies $(\bar{\omega} = \omega a^2/h(\rho/Y_2)^{1/2})$ of square sandwich plate with a central square delamination	126
6.5	Natural frequencies (Hz) of square sandwich plate with multiple delaminations .	133
6.6	Nondimensional natural frequencies $(\bar{\omega} = \omega a^2/h(\rho/Y_2)^{1/2})$ of simply supported composite spherical shell with a cutout and delamination around it	136
6.7	Nondimensional natural frequencies $(\bar{\omega} = \omega a^2/h(\rho/Y_2)^{1/2})$ of clamped composite spherical shell with a cutout and delamination around it	137
6.8	Natural frequencies (Hz) of simply supported sandwich cylindrical shell with multiple delaminations $(R/a=1)$	138
6.9	Natural frequencies (Hz) of clamped sandwich cylindrical shell with multiple delaminations $(R/a=1)$	140
7.1	Material properties	147
7.2	Natural frequencies (Hz) of cantilever aluminum plate with actuators in healthy and partially debonded conditions	148
7.3	Natural frequencies (Hz) of cantilever smart composite cylindrical shell with delamination and partial debonding in actuators and sensors	150
7.4	Natural frequencies (Hz) of cantilever smart composite cylindrical shell with delamination and/or partial debonding in actuators and/or sensors	152

7.5 Natural frequencies (Hz) of simply supported smart sandwich plate integrated with patch actuator and sensor and featuring multiple delaminations 154

7.6 Natural frequencies (Hz) of simply supported smart sandwich plate integrated with patch actuator and sensor and featuring single and multiple delaminations ($\bar{S} = 100$) 157

7.7 Natural frequencies (Hz) of cantilever smart composite plate with delamination and/or partial debonding in actuators and/or sensors 158

7.8 Settling time and peak control voltage for cantilever smart composite plate with delamination and transducer debonding under step load of 0.4 N 161

7.9 Settling time and peak control voltage for cantilever smart composite plate with delamination and transducer debonding under impulse load of 0.0075 N-s 169

7.10 Natural frequencies (Hz) of cantilever smart composite shell with delamination and/or partial debonding in actuators and/or sensors 171

7.11 Settling time and peak control voltage for cantilever smart composite shell with delamination and transducer debonding under step load of 30 N 171

7.12 Settling time and peak control voltage for cantilever smart composite shell with delamination and transducer debonding under impulse load of 0.075 N-s 176