

**SOME STUDIES ON THE TIME-VARIANCE OF
TIME-VARYING SYSTEMS AND THE
NON-STATIONARITY OF THE ASSOCIATED PROCESSES**

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**DEPARTMENT OF ELECTRICAL
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by

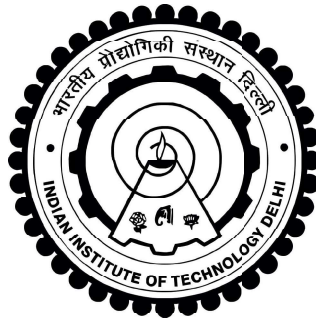
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भगवान् महागणपति
को समर्पित

Certificate

This is to certify that the thesis entitled “**Some studies on the time-variance of time-varying systems and the non-stationarity of the associated processes**” being submitted by **Mr. Ayan Kumar Dutta** to the Department of Electrical Engineering, Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy** is the record of the bonafide research work carried out by him under my supervision. In my opinion, the thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted either in part or in full to any other university or institute for the award of any degree or diploma.

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Abstract

Quantifying the degree of time-variance of a system and the degree of non-stationarity of a random process can help analyze, process, and synthesize signals and systems. In this thesis, we investigate the time-variability of linear periodically time-varying (LPTV) systems and the non-stationarity of wide sense cyclostationary (WSCS) random processes (with discrete-time input and output). We provide measures for the degree of time-variance of a system and the degree of non-stationarity of a random process. Whereas one measure for the degree of time-variance exploits the periodicity of the kernel and provides the measures in terms of Fourier coefficients, the other measure is computationally more efficient and provides the measure in terms of system kernel only. The second measure is also very general and can be used for various time-varying systems. We then consider the non-stationarity of WSCS random processes and, assuming that any WSCS process is the output of some LPTV system with white noise as input, provide a measure for the degree of non-stationarity of the process in terms of the departure of time-varying autocorrelation operator from the time-invariant part. In signal processing applications, multirate systems play an essential part, and we provide explicit expressions for the measures for various scenarios involving multi-rate systems.

We also propose a novel multi-rate system family that can periodically

introduce time-varying up-sampling rates. As we observe later, such a structure can model or generate semi-periodic signals like ECG, jitter, and voiced phonemes. This family is termed as DT-TVUS. Since this family of structures results in time-varying systems, one of the exciting applications of such a family could be to manipulate the non-stationarity of random processes. Hence, we deeply investigate the spectral properties of the output processes of DT-TVUS for various inputs, including WS-ACS, WSCS, and WSS. We also measure the degree of non-stationarity for a wider class of non-stationary processes.

Motivated by the range of applications that the proposed new family of structures can provide, we also propose a model for ECG signals and provide closed-form expressions for the time-varying autocorrelation function as well as the power spectral density of the process generated by the proposed model, under some assumptions on the model for analytical tractability.

सार

एक प्रणाली के समय विचरण की डिग्री और एक यादृच्छिक प्रक्रिया की गैर-स्थिरता की डिग्री को मापने से संकेतों और प्रणालियों का विश्लेषण, प्रक्रिया और संश्लेषण करने में मदद मिल सकती है। इस थीसिस में, हम रैखिक समय-परिवर्तनशील (एलपीटीवी) प्रणालियों की समय-परिवर्तनशीलता और व्यापक अर्थ साइक्लोस्टेशनरी (डब्ल्यूएससीएस) यादृच्छिक प्रक्रियाओं की गैर-स्थिरता (अलग-अलग समय इनपुट और आउटपुट के साथ) की जांच करते हैं। हम एक प्रणाली के समय-विचरण की डिग्री और एक यादृच्छिक प्रक्रिया की गैर-स्थिरता की डिग्री के लिए उपाय प्रदान करते हैं। जबकि समय-विचरण की डिग्री के लिए एक माप कर्नेल की आवश्यकता का फायदा उठाता है और फूरियर गुणांक के संदर्भ में उपाय प्रदान करता है, दूसरा माप कम्प्यूटेशनल रूप से अधिक कुशल है और केवल सिस्टम कर्नेल के संदर्भ में माप प्रदान करता है। दूसरा उपाय भी बहुत सामान्य है और इसका उपयोग विभिन्न समय-भिन्न प्रणालियों के लिए किया जा सकता है। फिर हम डब्ल्यूएससीएस यादृच्छिक प्रक्रियाओं की गैर-स्थिरता पर विचार करते हैं और यह मानते हुए कि कोई भी डब्ल्यूएससीएस प्रक्रिया इनपुट के रूप में सफेद शोर के साथ कुछ एलपीटीवी प्रणाली का आउटपुट है, प्रस्थान के संदर्भ में प्रक्रिया की गैर-स्थिरता की डिग्री के लिए एक उपाय प्रदान करते हैं। समय-अपरिवर्तनीय भाग से समय-भिन्न ऑटोसहसंबंध ऑपरेटर। सिग्नल प्रोसेसिंग अनुप्रयोगों में, मल्टीरेट सिस्टम एक आवश्यक भूमिका निभाते हैं, और हम मल्टी-रेट सिस्टम से जुड़े विभिन्न परिदृश्यों के उपायों के लिए स्पष्ट अभिव्यक्ति प्रदान करते हैं।

हम एक नवीन बहु-दर प्रणाली परिवार का भी प्रस्ताव करते हैं जो समय-समय पर समय-भिन्न अप-सैंपलिंग दरों को पेश कर सकता है। जैसा कि हमने बाद में देखा, ऐसी संरचना ईसीजी, जिटर और आवाज वाले स्वर जैसे अर्ध-आवधिक संकेतों को मॉडल या उत्पन्न कर

सकती है। इस परिवार को DT-TVUS कहा जाता है। चूंकि संरचनाओं का यह परिवार समय-भिन्न प्रणालियों में परिणत होता है, ऐसे परिवार के रोमांचक अनुप्रयोगों में से एक यादृच्छिक प्रक्रियाओं की गैर-स्थिरता में हेरफेर करना हो सकता है। इसलिए, हम WS-ACS, WSCS और WSS सहित विभिन्न इनपुट के लिए DT-TVUS की आउटपुट प्रक्रियाओं के वर्णक्रमीय गुणों की गहराई से जांच करते हैं। हम गैर-स्थिर प्रक्रियाओं के व्यापक वर्ग के लिए गैर-स्थिरता की डिग्री को भी मापते हैं।

संरचनाओं के प्रस्तावित नए परिवार द्वारा प्रदान किए जा सकने वाले अनुप्रयोगों की श्रृंखला से प्रेरित होकर, हम ईसीजी संकेतों के लिए एक मॉडल भी प्रस्तावित करते हैं और समय-भिन्न ऑटोसहसंबंध फ़ंक्शन के साथ-साथ उत्पन्न प्रक्रिया की शक्ति वर्णक्रमीय घनत्व के लिए बंद-रूप अभिव्यक्ति प्रदान करते हैं। प्रस्तावित मॉडल, विश्लेषणात्मक ट्रैक्टेबिलिटी के लिए मॉडल पर कुछ मान्यताओं के तहत।

Contents

Certificate	i
Acknowledgements	ii
Abstract	iii
सार	v
List of Figures	xi
List of Tables	xiii
1 Introduction	1
1.1 Perspective	1
1.2 Motivation and Literature Survey	2
1.3 Contribution of the Thesis	6
1.4 Outline of the thesis	7
2 Preliminaries and Related Works	11
2.1 Introduction	11
2.2 Multirate Filter Banks	11
2.2.1 Fundamentals	11
2.2.2 Polyphase Decomposition	12

2.3	Linear Time-Varying System	13
2.4	WSS/WSCS Process	14
2.4.1	Motivation	14
2.4.2	Some Relevant Definitions Related to Random Processes	14
3	Time-Variance and Non-stationarity Measures for Discrete LPTV Systems: A Theoretical Perspective	17
3.1	Introduction	17
3.2	Notations and Preliminaries	19
3.3	Measure for Degree of Time-Variance	20
3.4	Measure for Degree of Non-Stationarity	24
3.5	Conclusion	28
4	ℓ_2 and ℓ_∞ Measures for the Degree of Time-Variance in the Context of Filter Banks	29
4.1	Introduction	29
4.2	Notation and Preliminaries	32
4.3	Methodology	33
4.3.1	LPTV System Representation	34
4.3.2	Closest LTI system	39
4.4	Results and Simulations	42
4.4.1	Single Channel (SC)	42
4.4.2	Uniform Filter Bank (UFB)	44
4.4.3	Non-Uniform Filter Bank (NUFB)	49
4.4.4	Equivalent UFB from NUFB	51
4.4.5	Comparison Results	52
4.4.6	Uniform Filter Bank (UFB)	53
4.5	Non Uniform Filter Bank (NUFB)	54

4.6	Application	55
4.7	Conclusions	56
5	On a Novel Time-Varying Upsampling Rate Structure	60
5.1	Introduction	60
5.2	Notations and Preliminaries	64
5.3	Measure for the Degree of Time-Variance for DT-TVUS – Q System	64
5.4	Spectral Analysis	66
5.4.1	Background and Motivation	66
5.5	Conclusion	69
6	A Novel Model for ECG Signal and its Statistical Properties	71
6.1	Introduction	71
6.2	Notations and Preliminaries	74
6.3	Proposed ECG Model	76
6.3.1	Time-varying mean of the proposed model	76
6.3.2	Time-varying covariance of the proposed model	77
6.3.3	Time-varying spectrum of the proposed model	82
6.4	Degree of Non-stationarity	85
6.5	Results and Discussion	86
6.6	Conclusions	88
7	Conclusions and Future Scope of the Work	90
7.1	Salient conclusions from the thesis	90
7.2	Future Scope	91
	Bibliography	93

List of Publications	116
Biography of Author	117

List of Figures

3.1	Representation of a linear single-input single-output (SISO) system, with kernel $h(n, r)$	20
3.2	Conceptual diagram involving H, J_{cl}, e, Θ	21
4.1	SISO LPTV system $\mathcal{G}$ having kernel $g[m, r]$	34
4.2	MIMO representation.	34
4.3	Advancement followed by downsampler.	35
4.4	Shifted Samples.	35
4.5	MIMO LTI system representation.	38
4.6	Diagram illustrating the relationship between $g(m, r)$ and $\tilde{g}_{u,r}(d_1)$	40
4.7	Single channel of a filter bank [57].	43
4.8	$\text{MDTV}_{sc}^{\ell_2}$ for various low pass filters (LPFs). The coefficients for the various LPFs have been taken from [93].	44
4.9	Plot of $\text{MDTV}_{ufb}^{\ell_2}$ for various sets of low pass filters (LPFs). The coefficients for the different LPFs were obtained from [93].	45
4.10	Uniform Filter Bank (UFB) structure [93].	47
4.11	Non-uniform filter bank (NUFB) [78].	58
4.12	Equivalent UFB [102]	58
4.13	Perfect Reconstruction Filter Bank (PRFB) having $\text{MDTV}_{\text{Case 1}}^{\ell_2} = 0$	59

4.14	$B_0(z)$ replaced with $\tilde{B}_0(z)$. Here, $\text{MDTV}_{\text{Case } 2}^{\ell_2} = 0.0349 \neq 0$. .	59
4.15	$B_0(z)$ replaced with $\tilde{B}_0(z)$ and $B_1(z)$ replaced with $\tilde{B}_1(z)$. Here, $\text{MDTV}_{\text{Case } 3}^{\ell_2} = 0.1335 \neq 0$	59
6.1	A schematic representation of our proposed model experimented upon a Normal sinus rhythm.	75

List of Tables

4.1	Values of $\text{MDTV}_{sc}^{\ell_2}$, $\text{MDTV}_{sc}^{\ell_\infty}$, $\text{MDTV}_{ufb}^{\ell_2}$, $\text{MDTV}_{ufb}^{\ell_\infty}$ for various low pass filters (LPFs). [93] provides the coefficients for the various LPFs. Note that filters in all channels of UFB are identical and $L = 5$	46
4.2	Values of $\text{MDTV}_{ufb}^{\ell_2}$ and $\text{MDTV}_{ufb}^{\ell_\infty}$ for distinct low pass filters (LPFs) in distinct channels of UFB. The coefficients for the different LPFs were obtained from [93].	48
4.3	MDTV^{ℓ_2} and $\text{MDTV}^{\ell_\infty}$ for QMF / CQF filter banks.	48
4.4	$\text{MDTV}_{nufb}^{\ell_2}$ and $\text{MDTV}_{conv}^{\ell_2}$ values for non-compatible set : $L = 3$, $P_0 = 2$, $P_1 = 3$, $P_2 = 6$ and compatible set : $L = 5$, $P_0 = 2$, $P_1 = 4$, $P_2 = 6$, $P_3 = 24$, $P_4 = 24$. The various low pass filter coefficients have been obtained from [93]. Information regarding compatible and non-compatible sets is available in [77].	50
4.5	$\text{MDTV}_{nufb}^{\ell_\infty}$ and $\text{MDTV}_{conv}^{\ell_\infty}$ values for non-compatible set : $L = 3$, $P_0 = 2$, $P_1 = 3$, $P_2 = 6$ and compatible set : $L = 5$, $P_0 = 2$, $P_1 = 4$, $P_2 = 6$, $P_3 = 24$, $P_4 = 24$. The various low pass filter coefficients have been obtained from [93].	51
6.1	MDNST for Normal Sinus Rhythm Persons-1,2	87
6.2	MDNST for Normal Sinus Rhythm Persons-3,4	87

6.3	MDNST for Normal Sinus Rhythm Persons-5,6	87
6.4	MDNST for Sleep Apnea Persons-1,2	87
6.5	MDNST for Sleep Apnea Persons-3,4	88
6.6	MDNST for Sleep Apnea Persons-5,6	88
6.7	MDNST for Supraventricular Arrhythmia Patients-1,2	88
6.8	MDNST for Supraventricular Arrhythmia Patients-3,4	88
6.9	MDNST for Supraventricular Arrhythmia Patients-5,6	89