# FINITE ELEMENT ANALYSIS OF ELASTIC CONTACT PROBLEM AND ITS APPLICATION FOR THE STUDY OF THE EFFECT OF JOINT FLEXIBILITY ON FREE VIBRATION CHARACTERISTICS OF MACHINE TOOL STRUCTURES

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CERTIFICATE

This is to certify that the thesis entitled 'Finite Element Analysis of Elastic Contact Problem and its Application for the Study of the Effect of Joint Flexibility on Free Vibration Characteristics of Machine Tool Structures' being submitted by Tulsi Dass Sachdeva to the Indian Institute of Technology, Delhi, India, for the award of the degree of Doctor of Philosophy in Applied Mechanics is a record of bonafide research work carried out by him under our supervision and guidance. The thesis work in our opinion has reached the standard fulfilling the requirements for the Doctor of Philosophy Degree. The research report and the results presented in this thesis have not been submitted in part or in full to any other University or Institute for the award of any degree or diploma.

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## ABSTRACT

This study is concerned with the development of procedure for computing the flexibilities of dovetail joint and studying the effect of the joint flexibility on the free vibration characteristics of a milling machine structure. The joint flexibility has been modelled with joint elements, the stiffness matrix of which is compatible with beam elements. For studying the vibration characteristics, the milling machine has been modelled as an assemblage of beam elements with consistent mass matrix.

The inplane flexibilities of the dovetail joint have been computed, treating it as an elastic contact problem using the finite element method. Algorithms and computer programs have been developed to solve the elastic contact problem with force boundary conditions for the frictionless contact and the contact with friction. The effect of friction on the inplane flexibilities of a dovetail joint has been studied.

For computing the out-of-plane flexibilities, the semi-analytical FE method combining the Fourier Series expansion with 2-D elastic analysis has been used. A reasonable distribution of contact pressure has been assumed along the depth of the joint.

In all the above studies, eight noded quadratic isoparametric elements have been used and a state of plane strain has been assumed for computing the inplane

flexibilities. A 2 x 2 Gaussian integration has been used and the solution of equations is obtained with Frontal House-keeping algorithm.

For the knee-column joint and the overarm-column joint, deformed shapes have been computed by keeping one end of joint fixed and applying the loads at the other end. From the flexibilities obtained for one end of the joint, the 12 x 12 stiffness matrix, compatible with beam elements has been computed.

The natural frequencies and mode shapes of a milling machine have been computed by modelling it with beam elements. The generalized eigenvalue problem has been solved by the Sub-polynomial Iteration Method and the Determinant Search Method. Frontal House-keeping algorithm has been used in the computer programs developed for the eigenvalue problem. Computations have been carried out by assuming rigid joints and also for the case when the flexibility of the joints is taken into account. It is observed that the overarm column joint has a marked effect on the free vibration characteristics of the milling machine structure.

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