MITIGATING RISK WITH PORTFOLIO OPTIMIZATION: ENHANCED INDEXING AND ROBUST PERSPECTIVE

RUCHIKA SEHGAL

DEPARTMENT OF MATHEMATICS
INDIAN INSTITUTE OF TECHNOLOGY DELHI

OCTOBER 2020

©Indian Institute of Technology Delhi (IITD), New Delhi, 2020

MITIGATING RISK WITH PORTFOLIO OPTIMIZATION: ENHANCED INDEXING AND ROBUST PERSPECTIVE

by

Ruchika Sehgal

Department of Mathematics

submitted
in fulfillment of the requirements of the degree of Doctor of Philosophy

to the

INDIAN INSTITUTE OF TECHNOLOGY DELHI

OCTOBER 2020

Dedicated To My Family

Certificate

This is to certify that the thesis entitled **Mitigating Risk with Portfolio Optimization: Enhanced Indexing and Robust Perspective** submitted by **Ms. Ruchika Sehgal**, to the Indian Institute of Technology Delhi, for the award of the Degree of **Doctor of Philosophy**, is a record of the original bonafide research work carried out by her under my supervision. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted in part or full to any other university or institute for the award of any degree or diploma.

Dr. Aparna Mehra

Professor

Department of Mathematics

Indian Institute of Technology Delhi

Date: October 12, 2020

New Delhi 110 016

Acknowledgements

First and foremost, I thank God for giving me the patience and the capability to successfully complete my thesis.

This thesis is a culmination of support and guidance from several remarkable individuals who encouraged me to go through this journey patiently.

I wish to express my sincere thanks to my supervisor Prof. Aparna Mehra for providing me the opportunity to pursue my doctoral studies under her supervision. I will be eternally grateful to her for the expert advice, perennial support, and patience that helped me grow professionally. I have learnt a lot from her in the last few years and I truly thank her for her new ideas, fruitful discussions, and moral support throughout my doctoral studies.

I am thankful to my SRC members: Prof B.S. Panda, Prof S. Dharmaraja, and Prof. S.S. Yadav for their time and encouraging comments to improve this thesis. I am also deeply thankful to all expert referees and editors of Journals for evaluating my research papers which significantly contributed to improve the quality of research.

I acknowledge IIT Delhi for providing me GATE fellowship and travel grants for my doctoral studies. I am thankful to IIT Delhi authorities, hostel staff, administration, and security for providing the necessary facilities and a safe environment. I am indebted to IIT Delhi for providing access to Thomson Reuters Eikon software in the Mathematics Department that made it extremely for me to access financial data for research.

I deeply acknowledge all my teachers who taught me at different stages of my life and inspired me to pursue this career. A special mention goes to Dr. William Mcghee and Ms. Ruchita Gomkale for the opportunity to work on a project at Royal Bank of Scotland under their supervision. The project enabled me to explore beyond my research area and provided me exposure to the corporate culture. I acknowledge Haryana higher education department for the job opportunity as an assistant professor. I am thankful to Ms. Namarta Sharma, principal of my college, for extending her cooperation.

I express my heartfelt thanks to my seniors Dr. Amita Sharma, Dr. Anubha Goel, and Dr. Puneet Pasricha for their helpful nature and insightful guidance during my research. I feel blessed to have friends like Juhi, Lipsy, Manpreet, Navneet, Anuj, Lalit, Aman, Divya, Abhay, and Kartikay. They have been great companions and I will cherish their friendship forever. I would especially like to thank Dr. Dileep Kumar for being with me during both joyous and low moments of our journey at IIT Delhi. He has been my pillar of strength and a source of happiness since the time we met.

I would like to dedicate this thesis to my family for their love and constant support. My mother is very caring, loving, and worried for me throughout my doctoral studies. This thesis would not have been possible without her selfless love and support. My father is patient, understanding, and encouraging. A special thanks goes to my sister Sheetal Sehgal, brother-in-law Rohit Choudhry and my lovely nephew Arjun for their love and affection.

October 2020 — *Ruchika Sehgal*

Abstract

An investor aims to achieve high yields when investing in risky assets. The primary concern of an investor is to select assets that meet his return-risk requirement. The number of investment opportunities has risen to a considerable extent over the past two decades. As a result, financial markets across the globe have experienced spiraling upward growth in recent decades, with strong capital inflows. It is, therefore, becoming more critical for fund managers and investors to pick the assets for portfolio formation carefully. Diversification provides low-risk exposure as opposed to the risk of investing in a single asset or a group of a handful of assets. It helps to reduce volatility in the portfolio returns, which is mainly required when returns of some assets in the portfolio experience a sharp drop.

Portfolio optimization is a process of constructing a portfolio by splitting the capital to be invested optimally based on the return-risk profile of the investor. The present thesis proposes optimization models that aim to mitigate risk in the portfolio, where different risk metrics applied to measures risk. We primarily study two types of portfolio optimization models, namely enhanced indexing and robust portfolio selection. The key focus of the proposed models is on minimizing the risk and analyzing the trade-off between the expected return and its associated risk. We have carried out an extensive empirical analysis of all the proposed models on the financial data sets taken from a variety of markets across the globe. We adopted the in-sample and out-of-sample approach in the empirical experiments, where we have used the return data of the in-sample period for constructing the portfolio and used the out-of-sample return data to test the viability of the portfolio.

In Chapter 2, we propose an enhanced indexing portfolio optimization model that not only seeks to maximize the excess returns over and above the benchmark index but simultaneously control the risk by introducing a constraint on the weighted conditional value at risk of the portfolio. The constraint in the proposed model hedges the risk described by weighted conditional value at risk of the portfolio.

In Chapter 3, we present an enhanced indexing model that attempts to outperform the benchmark index by aiming at a specific quantile of return distribution plus some alpha return using quantile regression. The conditional value at risk metric in the model controls downside risk in a portfolio. Constraints on short-selling and portfolio rebalancing with transaction and holding costs are built in the models to integrate real-life functionalities. The proposed models are linear or mixed-integer linear programs, and hence computationally tractable.

Chapters 4-6 of the thesis focus on analyzing robustness in input parameters in portfolio optimization models. We address two types of uncertainty in the model: one that arises by the probability distribution of returns, and another is inherited in returns itself. Portfolio optimization models typically assume that each scenario is equally probable or follows uniform distribution. This conclusion, however, is not true to hold in reality. In our research, summarized in Chapter 4, we introduce the worst-case portfolio optimization models within the robust optimization framework for maximizing return through either the mean or median metrics. The risk in the portfolio is quantified by Gini mean difference. We put forward the worst-case models under the mixed and interval+polyhedral uncertainty sets. The uncertainty is considered in probability distribution followed by return of the portfolio.

In Chapters 5 and 6, we concentrate on modeling the uncertainty in the returns itself. There are quite a few reasons to research this kind of uncertainty. The out-of-sample returns of an asset may widely vary from its in-sample returns. The portfolio optimization problem, when solved with the historical in-sample returns (construction period), may not yield a portfolio that performs well in the out-of-sample period (investment period). Moreover, the formulated optimization models require to note the returns from market at discrete time points. Generally, we use the closing prices of the asset on a daily or weekly basis to compute its return. Regardless of the time for computing return, some vital information is lost by not taking into account the intra-day/intra-week variations in return. To address these concerns, it is but natural to consider uncertainty in returns of an asset in the in-sample period of the portfolio selection. In this thesis, we address the uncertainty in returns of assets by varying them in symmetric bounded intervals.

In Chapter 5, we propose the robust portfolio optimization models for reward-risk ratios utilizing Omega, semi-mean absolute deviation, and weighted STARR ratios.

In Chapter 6, we propose a robust portfolio optimization model involving second-order stochastic dominance in constraints. A portfolio optimization problem equipped with stochastic dominance constraints and the returns varying in the bounded intervals about their nominal values yields optimal robust portfolios ideal for rational and risk-averse investors.

Chapter 7 concludes the work with some future directions.

सारांश

एक निवेशक का उद्देश्य जोखिम भरी परिसंपत्तियों में निवेश करते समय उच्च रिटर्न प्राप्त करना है। एक निवेशक की प्राथमिक चिंता उन परिसंपत्तियों का चयन करना है जो उनकी वापसी-जोखिम आवश्यकता को पूरा करती हैं। पिछले दो दशकों में निवेश के अवसरों की संख्या काफी हद तक बढ़ी है। नतीजतन, दुनिया भर के वित्तीय बाजारों में हाल के दशकों में मजबूत पूंजी प्रवाह के साथ वृद्धि का अनुभव हुआ है। इसलिए, फंड प्रबंधकों और निवेशकों के लिए पोर्टफोलियो गठन के लिए परिसंपत्तियों को ध्यान से चुनना अधिक महत्वपूर्ण हो गया है। एकल परिसंपत्ति या मुट्ठी भर परिसंपत्तियों के समूह में निवेश के जोखिम के विपरीत विविधीकरण कम जोखिम प्रदान करता है। यह पोर्टफोलियो रिटर्न में अस्थिरता को कम करने में मदद करता है जब पोर्टफोलियो में कुछ परिसंपत्तियों के रिटर्न में तेज गिरावट का अनुभव होता है, जो मुख्य रूप से आवश्यक है।

पोर्टफोलियो ऑप्टिमाइज़ेशन निवेशक के रिटर्न-रिस्क प्रोफाइल के आधार पर बेहतर निवेश के लिए पूंजी को विभाजित करके एक पोर्टफोलियो के निर्माण की एक प्रक्रिया है। वर्तमान थीसिस ने ऑप्टिमाइज़ेशन मॉडल का प्रस्ताव किया है जिसका उद्देश्य पोर्टफोलियो में जोखिम को कम करना है, जहां जोखिम को मापने के लिए विभिन्न जोखिम मीट्रिक लागू होते हैं। हम मुख्य रूप से दो तरह के पोर्टफोलियो ऑप्टिमाइज़ेशन मॉडल का अध्ययन करते हैं, जिसका नाम है एनहांस्ड इंडेक्सिंग और रोबस्ट पोर्टफोलियो चयन। प्रस्तावित मॉडल का मुख्य फोकस जोखिम को कम करने और अपेक्षित रिटर्न और इससे जुड़े जोखिम के बीच व्यापार-बंद का विश्लेषण करना है। हमने दुनिया भर के विभिन्न बाजारों से लिए गए वित्तीय डेटा सेटों पर सभी प्रस्तावित मॉडलों का व्यापक अनुभवजन्य विश्लेषण किया है। हमने अनुभवजन्य प्रयोगों में इन-सैंपल और आउट-ऑफ-सैंपल दृष्टिकोण को अपनाया, जहां हमने पोर्टफोलियो के निर्माण के लिए इन-सैंपल अवधि के रिटर्न डेटा का उपयोग किया है और पोर्टफोलियो की व्यवहार्यता का परीक्षण करने के लिए आउट-ऑफ-सैंपल रिटर्न डेटा का उपयोग किया है।

अध्याय 2 में हम एक एनहांस्ड इंडेक्सिंग पोर्टफोलियो ऑप्टिमाइज़ेशन मॉडल का प्रस्ताव करते हैं जो न केवल बेंचमार्क इंडेक्स के ऊपर और अधिक से अधिक रिटर्न को अधिकतम करने का प्रयास करता है, साथ ही साथ पोर्टफोलियो के वेटिड कंडिशनल वैल्यू एट रिस्क पर एक कनस्ट्रेंट से जोखिम को नियंत्रित करता है। प्रस्तावित मॉडल में पोर्टफोलियो के वेटिड कंडिशनल वैल्यू एट रिस्क जोखिम को कम करती है।

अध्याय 3 में हम क्वांटाइल रिग्रेशन का उपयोग करके एक एनहांस्ड इंडेक्सिंग पोर्टफोलियो ऑप्टिमाइज़ेशन मॉडल प्रस्तुत करते हैं जो रिटर्न वितरण के एक विशिष्ट क्वांटाइल और कुछ अल्फा रिटर्न में लक्ष्य करके बेंचमार्क इंडेक्स को बेहतर बनाने का प्रयास करता है। कंडिशनल वैल्यू एट रिस्क पोर्टफोलियो में नकारात्मक जोखिम को नियंत्रित करता है। शॉर्ट-सेलिंग और पोर्टफोलियो रिबैलेंसिंग पर लेनदेन और होल्डिंग लागत के साथ वास्तविक मॉडल को वास्तविक जीवन में एकीकृत करने के लिए

मॉडल में बनाया गया है। प्रस्तावित मॉडल रैखिक या मिश्रित-पूर्णांक रैखिक प्रोग्रैम हैं, और इसलिए कम्प्यूटेशनल रूप से ट्रैक्टेबल हैं।

थीसिस के 4-6 अध्याय पोर्टफोलियो ऑप्टिमाइज़ेशन मॉडल में इनपुट मापदंडों में रोबस्टनेस का विश्लेषण करने पर ध्यान केंद्रित करते हैं। हम मॉडल में दो प्रकार की अनिश्चितता को संबोधित करते हैं: एक जो रिटर्न की संभावना वितरण से उत्पन्न होता है, और दूसरा रिटर्न में ही विरासत में मिलता है। पोर्टफोलियो ऑप्टिमाइज़ेशन मॉडल आमतौर पर मानते हैं कि प्रत्येक परिदृश्य समान रूप से संभावित है या समान वितरण का अनुसरण करता है। हमारे शोध में अध्याय 4 में संक्षेप में हम औसत या औसतन मेट्रिक्स के माध्यम से रिटर्न को अधिकतम करने के लिए मजबूत अनुकूलन ढांचे के भीतर सबसे खराब स्थिति वाले पोर्टफोलियो अनुकूलन मॉडल का परिचय देते हैं। पोर्टफोलियो में जोखिम को गिनी मीन डिफेरेंस द्वारा निर्धारित किया जाता है। हमने मिश्रित और ईटरवल + पॉलीहेड्रल अनिश्चितता सेट के तहत सबसे खराब स्थिति वाले मॉडल को आगे रखा। पोर्टफोलियो में रिटर्न के संभाव्यता वितरण में अनिश्चितता पर विचार किया जाता है।

अध्याय 5 और 6 में हम रिटर्न में अनिश्चितता के बारे में मॉडलिंग करने पर ध्यान केंद्रित करते हैं। इस तरह की अनिश्चितता पर शोध करने के लिए कुछ कारण हैं। किसी परिसंपत्ति का आउट-ऑफ-सैंपल रिटर्न व्यापक रूप से इसके इन-सैंपल रिटर्न से भिन्न हो सकता है। पोर्टफोलियो ऑप्टिमाइज़ेशन समस्या जब ऐतिहासिक इन-सैंपल रिटर्न (निर्माण अवधि) के साथ हल किया जाता है, तो एक पोर्टफोलियो नहीं मिल सकता है जो आउट-ऑफ-सैंपल अवधि (निवेश अवधि) में अच्छा प्रदर्शन करता है। इसके अलावा, तैयार ऑप्टिमाइज़ेशन मॉडल असतत समय बिंदुओं पर बाजार से रिटर्न नोट करने की आवश्यकता है। आमतौर पर, हम इसकी वापसी की गणना करने के लिए दैनिक या साप्ताहिक आधार पर संपत्ति की समापन कीमतों का उपयोग करते हैं। रिटर्न को किसी भी समय लिय जाने के बावजूद कुछ महत्वपूर्ण जानकारी बदले में इंट्रा-डे / इंट्रा-सप्ताह विविधताओं को ध्यान में नहीं रखकर खो जाती है। इन चिंताओं को दूर करने के लिए पोर्टफोलियो चयन के इन-सैंपल अवधि में किसी संपत्ति के रिटर्न में अनिश्चितता पर विचार करना स्वाभाविक है। इस थीसिस में हम परिसंपत्तियों के रिटर्न में अनिश्चितता को सममित बाउंड अंतराल में अलग-अलग करके संबोधित करते हैं।

अध्याय 5 में हम ओमेगा, सेमि मीन डिफेरेंस और वेटिड स्टार अनुपात का उपयोग करते हुए रीवॉर्ड-रिस्क अनुपात के लिए मजबूत पोर्टफोलियो ऑप्टिमाइज़ेशन मॉडल का प्रस्ताव करते हैं।

अध्याय 6 में हम एक रोबस्ट पोर्टफोलियो ऑप्टिमाइज़ेशन मॉडल का प्रस्ताव करते हैं जिसमें बाधाओं में दूसरे क्रम के स्टोकेस्टिक डोमिनेंस शामिल हैं। स्टोकेस्टिक डोमिनेंस बाधाओं से लैस एक पोर्टफोलियो ऑप्टिमाइज़ेशन समस्या और उनके नाममात्र मूल्यों के बारे में बंधे अंतराल में भिन्न रिटर्न तर्कसंगत और जोखिम-प्रतिकूल निवेशकों के लिए आदर्श मजबूत पोर्टफोलियो प्राप्त करता है।

अध्याय 7 भविष्य के कुछ निर्देशों के साथ थीसिस का निष्कर्ष निकालता है।

Table of Contents

Certificate **i**

Acknowledgements **iii**

Abstract **v**

List of Figures **x**

List of Tables **xii**

List of Abbreviations **xvii**

1 Introduction **1**

1.1 Literature Survey . . . 3
1.1.1 Return-Risk Models . . . 3
1.1.2 Stochastic Dominance . . . 4
1.1.3 Index Tracking and Enhanced Indexing . . . 6
1.1.4 Robust Portfolio Optimization . . . 7
1.1.5 Quantile Regression . . . 9
1.2 Notations . . . 9
1.3 Existing Models in the Literature . . . 10
1.3.1 Weighted CVaR Model . . . 10
1.3.2 Gini Mean Difference Model . . . 12
1.3.3 The Median Portfolio Optimization Model . . . 13
1.3.4 Omega Ratio . . . 14
1.3.5 Semi-MAD Ratio . . . 15
1.3.6 Weighted STARR Ratio . . . 16
1.3.7 Second-order Stochastic Dominance Model . . . 17
1.3.8 Quantile Regression Model . . . 18
1.3.9 Enhanced Indexing Models in the Literature . . . 19
1.4 Performance Metrics . . . 21
1.5 Statistical Tests used in Analysis . . . 22
1.6 Analysis under Different Scenarios in Market . . . 23
1.7 Rolling Window Strategy . . . 23
1.8 Organization of the Thesis . . . 24

2 Enhanced Indexing using Weighted Conditional Value at Risk **27**

2.1 Motivation and Objective . . . 27
2.2 The Weighted CVaR Enhanced Indexing Model . . . 29
2.3 Data Sets and Methodology . . . 31
2.4 Out-of-Sample Analysis . . . 34
2.4.1 Single Window Analysis on First Data Set (C2 DI) . . . 34
2.4.2 Performance Analysis under Different Scenarios in Market . . . 40
2.4.3 Rolling Window Analysis on Data sets (C2 DII) - (C2 DV) . . . 42

3 Quantile Regression based Enhanced Indexing with Portfolio Rebalancing 51
3.1 Motivation and Objective . . . 51
3.2 Enhanced Indexing Models with Rebalancing . . . 52
3.3 Empirical Analysis . . . 54
3.3.1 Analysis without Short Selling . . . 55
3.3.2 Analysis with Short Selling . . . 59

4 Worst-case Analysis of Gini Mean Difference Safety Measure 65
4.1 Motivation and Objective . . . 65
4.2 Safety Measure Portfolio Optimization . . . 66
4.3 Robust Optimization with Mixed Uncertainty Set . . . 68
4.3.1 Mean-GMD Safety Model . . . 68
4.3.2 Median-GMD Safety Model . . . 69
4.4 Robust Optimization under Interval+Polyhedral Uncertainty Set . . . 70
4.4.1 Mean-GMD Safety Model . . . 70
4.4.2 Median-GMD Safety Model . . . 73
4.5 Empirical Analysis . . . 74
4.5.1 Mixture Distribution . . . 75
4.5.2 Interval+Polyhedral Uncertainty Set and Numerical Comparison of Uncertainty Sets . . . 84

5 Robust Reward-Risk Ratio Portfolio Optimization 87
5.1 Motivation and Objective . . . 87
5.2 Robust Reward-Risk Ratios . . . 88
5.2.1 Robust Omega Ratio . . . 89
5.2.2 Robust Semi-MAD Ratio . . . 92
5.2.3 Robust Weighted STARR Ratio . . . 92
5.3 Cutting Plane Algorithm . . . 93
5.4 Empirical Analysis . . . 95
5.4.1 Data sets . . . 95
5.4.2 Methodology . . . 95
5.4.3 In-Sample Analysis . . . 96
5.4.4 Out-of-Sample Analysis . . . 96

6 Robust Portfolio Optimization with Second Order Stochastic Dominance Constraints 107
6.1 Motivation and Objective . . . 107
6.2 Robust Second Order Stochastic Dominance Portfolio Optimization Model 108
6.3 Cutting Plane Algorithm . . . 111
6.4 Empirical Analysis . . . 112
6.4.1 Data sets and Methodology . . . 112
6.4.2 Time Efficiency of the Cutting Plane Algorithm . . . 114
6.4.3 The Out-of-Sample Analysis . . . 114

7 Conclusions, Limitations, and Future Work 119
7.1 Conclusions . . . 119

7.2 Limitations . . . 121
7.3 Future Works . . . 121

Bibliography **123**

Appendix **133**

Curriculum Vitae **135**

List of Figures

1.1 Rolling window strategy with N windows, T in-sample period, and T_1 out-of-sample period 24

2.1 Out-of-sample cumulative returns of optimal portfolios from $(GWCVaR)$, (MM), $(EWCVaR)$ and (SSD) models on data set (C2 DI) (a) S&P 500 for risk level U_2/K_2 (b) FTSE 100 for risk level U_3/K_3 38

2.2 Cumulative returns of S&P 500 index from November 1998-December 2005 depicting four scenarios in the market. 40

3.1 Out-of-sample weekly cumulative returns of optimal portfolios from the benchmark index (blue), QU_τ (red), and $\text{MEZ}_{\tau'}$ (purple) on data sets (a) (C3 DI) for $\tau = 0.8$ and $\tau^{'} = 0.2$ (b) (C3 DXII) for $\tau = 0.7$ and $\tau^{'} = 0.3$ (c) (C3 DXV) for $\tau = 0.75$ and $\tau^{'} = 0.25$. 58

3.2 Movements of market index (a) ASE (b) PSI-20 (c) S&P 500 (d) FTSE 100 during the respective periods of the financial turmoils as mentioned in the text. 62

4.1 Returns of market index S&P Asia 50 (C4 DIII) in one specific 27 weeks in-sample period from 13/10/2017 to 13/04/2018 to indicate different distributions followed by returns 75

4.2 Out-of-sample downside risk of (C4 DIII) for (a) (μGMD) and $(R\mu GMD)$ models and (b) $(MGMD)$ and $(RMGMD)$ models. 78

4.3 Cumulative returns of market indices corresponding to (C4 DI), (C4 DII), (C4 DIII), and (C4 DIV) depicting different phases of markets. 79

4.4 Out-of-sample cumulative returns of UP-DOWN phase of (C3 DIII) for (a) (μGMD) and $(R\mu GMD)$ model and (b) $(MGMD)$ and $(RMGMD)$ model. 80

4.5 Out-of-sample downside risk of UP-DOWN phase of $(C3DIII)$ for (a) (μGMD) and $(R\mu GMD)$ model and (b) $(MGMD)$ and $(RMGMD)$ model. 81

5.1 Trendline fitting in FTSE 100 from 15/01/2018 to 13/05/2019. 99

6.1 Out-of-sample downside risk of optimal SSD, $RP_{0.001}$, $RP_{0.005}$ and $RP_{0.01}$ portfolios on data set (a) (C6 DI) (b) (C6 DIV). 116

List of Tables

2.1 Tolerance sets and corresponding weights used in analyzing ($GWCVaR$) model. 34
2.2 Out-of-sample average returns ($\times 10^{-3}$) for a single window analysis on data set (C2 DI). 36
2.3 Out-of-sample performance ratios ($\times 10^{-3}$) for a single window analysis on data set (C2 DI), where the first and second sub-columns record SR and UPR, respectively; '-' is used for instances where SR is not defined. 37
2.4 Out-of-sample Weighted STARR ratio ($\times 10^{-3}$) and STARR ratio ($\times 10^{-3}$) for a single window analysis on data set (C2 DI), where the first, second, and third sub-columns represent Weighted STARR_{S1}, Weighted STARR_{S2} and $\text{STARR}_{0.1}$, respectively; '-' is used in instances where the Weighted STARR ratio or STARR ratio is not defined. 39
2.5 Out-of-sample statistics ($\times 10^{-3}$) for different phases of market in data set (C2 DII). 41
2.6 Out-of-sample cumulative returns ($\times 10^{-4}$) of optimal portfolios from ($GWCVaR$) model with tolerance level set S3 and corresponding ($CVaR_\gamma$) on data set (C2 DII). 43
2.7 Out-of-sample cumulative returns ($\times 10^{-4}$) of optimal portfolios from ($GWCVaR$) model with tolerance level set S2 and corresponding ($CVaR_\gamma$) on data set (C2 DII). 44
2.8 Out-of-sample cumulative returns ($\times 10^{-4}$) of optimal portfolios from ($GWCVaR$) model with tolerance level set S1 and corresponding ($CVaR_\gamma$) on data set (C2 DII). 45
2.9 Out-of-sample cumulative returns ($\times 10^{-4}$) of optimal portfolios from ($GWCVaR$) model with tolerance level set S3 and corresponding ($CVaR_\gamma$) on data set (C2 DIII). 46
2.10 Out-of-sample cumulative returns ($\times 10^{-4}$) of optimal portfolios from ($GWCVaR$) model with tolerance level set S2 and corresponding ($CVaR_\gamma$) on data set (C2 DIII). 47
2.11 Tolerance sets and corresponding weights used in analyzing ($WCVaR$) model. 47
2.12 Out-of-sample cumulative returns ($\times 10^{-4}$) of optimal portfolios from ($WCVaR$) model with tolerance level set S3 and corresponding ($CVaR_\gamma$) on (C2 DIV). 48
2.13 Out-of-sample cumulative returns ($\times 10^{-4}$) of optimal portfolios from ($WCVaR$) model with tolerance level set S2 and corresponding ($CVaR_\gamma$) on (C2 DV). 49
2.14 Out-of-sample cumulative returns ($\times 10^{-4}$) of optimal portfolios from ($WCVaR$) model with tolerance level set S1 and corresponding ($CVaR_\gamma$) on (C2 DV). 49

3.1 Out-of-sample weekly average returns ($\times 10^{-3}$) for (C3 DI)-(C3 DXV). 57
3.2 Out-of-sample weekly SR ($\times 10^{-3}$) for (C3 DI)-(C3 DXV); '-' indicates that the ratio can not be calculated for the instance. 57
3.3 Out-of-sample weekly STARR ratio ($\times 10^{-3}$) for (C3 DI)-(C3 DXV); '-' indicates that the ratio can not be calculated for the instance. 57
3.4 Out-of-sample weekly UPR ($\times 10^{-3}$) for (C3 DI)-(C3 DXV). 58

3.5 p values of t-test at significance levels 0.01, 0.05 and 0.1, indicated by ***, ** and *, respectively, for optimal portfolios from the $(QUEI)$ model with $\tau = 0.7,\ 0.75,\ 0.8$ against the benchmark index for data sets (C3 DI)-(C3 DXV). 58
3.6 Out-of-sample daily statistics ($\times 10^{-3}$) of S&P 500 under different conditions. 59
3.7 Out-of-sample weekly statistics ($\times 10^{-3}$) for (C3 SDI)-(C3 SDIV). 60
3.8 p values of t-test with significance levels 0.01, 0.05 and 0.1, indicated by ***, ** and *, respectively, for optimal portfolios from the $(SQUEI)$ and $(QUEI)$ model with $\tau = 0.7,\ 0.75,\ 0.8$ against the benchmark index for data sets (C3 SDI)-(C3 SDIV). 61
3.9 Out-of-sample weekly average returns ($\times 10^{-3}$) for (C3 CSDI)-(C3 CSDIV). 62
3.10 p values of t-test with significance levels 0.01, 0.05, and 0.1, indicated by ***, ** and *, respectively, for optimal portfolios from the $(SQUEI)$ and $(QUEI)$ model with $\tau = 0.7,\ 0.75,\ 0.8$ against the benchmark index for (C3 CSDI)-(C3 CSDIV). 63

4.1 Statistics of returns of market index S&P Asia 50 from 13/10/2017 to 13/04/2018 76
4.2 Out-of-sample statistics($\times 10^{-3}$) of (C4 DI), (C4 DII), (C4 DIII), and (C4 DIV) on rolling window analysis under mixed uncertainty set Q_M. 77
4.3 Out-of-sample statistics($\times 10^{-3}$) of (C4 DI) and (C4 DII) under mixed uncertainty set P_M for the four phases data based on the market directions. 82
4.4 Out-of-sample statistics($\times 10^{-3}$) of (C4 DIII) and (C4 DIV) under mixed uncertainty set P_M for the four phases data based on the market directions. 83
4.5 Out-of-sample statistics ($\times 10^{-3}$) of (C4 DI), (C4 DII), (C4 DIII) and (C4 DIV) on a single window analysis under mixed and interval+polyhedral uncertainty sets. 85

5.1 Time (in seconds) taken to solve $(ROMEGA)$ model by the cutting plane algorithm and the standard LP solver on (C5 DIII) where '$-$' indicates that the LP solver fails to obtain the optimal solution of the problem after running for 45 minutes. 97
5.2 Optimal values of $(ROMEGA)$ model. Note that for $\Gamma = 0$, the optimal values of $(ROMEGA)$ and $(OMEGA)$ models are the same, for any value of $\hat{r}$. 98
5.3 Market phase of FTSE 100 using trendline (dates in dd/mm/yyyy format) 99
5.4 The out-of-sample statistics ($\times 10^{-3}$) when $\Gamma = 5$ for data sets (C5 DI), (C5 DII), (C5 DIII), and (C5 DIV). Here, $(ROMEGA1)$ and $(ROMEGA2)$ are the robust Omega models for $\hat{r} = 0.1\%$ and $\hat{r} = 1\%$, respectively. Similar nomenclatures are followed for the other two models as well. 101
5.5 The out-of-sample statistics ($\times 10^{-3}$) when $\Gamma = 10$ for data sets (C5 DI), (C5 DII), (C5 DIII), and (C5 DIV). Here, $(ROMEGA1)$ and $(ROMEGA2)$ are the robust Omega models for $\hat{r} = 0.1\%$ and $\hat{r} = 1\%$, respectively. Similar nomenclatures are followed for the other two models as well. 102
5.6 The F_{σ^2} statistic (the p values are in parenthesis at significance levels 0.01, 0.05 and 0.1 indicated by ***, ** and *, respectively) for optimal portfolios from the non-robust and robust models with $\hat{r} = 1\%$. 103

5.7 The $z_{CVaR_{0.1}}$ statistic (the p values are in parenthesis at significance levels 0.01, 0.05 and 0.1, indicated by ***, ** and *, respectively) for optimal portfolios from the non-robust and robust models with $\hat{r} = 1\%$. 103
5.8 Out-of-sample statistics ($\times 10^{-3}$) under different market phases. 104
5.9 Out-of-sample statistics ($\times 10^{-3}$) under different market trends. 105

6.1 Time efficiency of the cutting plane algorithm; all times are in seconds and '-' indicates out of memory message on MATLAB. 114
6.2 Out-of-sample statistics ($\times 10^{-3}$) from the rolling window strategy at $\Gamma = 5$ for data sets (C6 DI)-(C6 DV). 115
6.3 Test values (with p values in bracket at significance level 0.1, 0.05 and 0.01 indicated by *, **, and *** respectively, for $SSD\text{-}R_{0.01}$). 116
6.4 Out-of-sample statistics ($\times 10^{-3}$) under different market phases. 117
6.5 Out-of-sample statistics ($\times 10^{-3}$) under different market trend with best values highlighted in bold. 118

List of Abbreviations

Concepts:

EI	Enhanced Indexing
IT	Index Tracking
QR	Quantile Regression
RO	Robust Optimization
SD	Stochastic Dominance
SSD	Second order Stochastic Dominance

Models:

LP	Linear Program
MILP	Mixed Integer Linear Program
QPP	Quadratic Programming Problem
SDP	Semi-definite Program
SOCP	Second Order Cone Program

Performance Metrics:

CVaR_γ	Conditional Value-at-Risk at tolerance level γ
MAD	Mean Absolute Deviation
SDV	Standard Deviation
SMAD	Semi Mean Absolute Deviation
SR	Sharpe Ratio
STARR	The Stable Tail-adjusted Return Ratio
UPR	Upside Potential Ratio
VaR_γ	Value-at-Risk at tolerance level γ
VAS	Violation area in Second order Stochastic Dominance
$\text{WCVaR}_{\{\gamma_1,\ldots,\gamma_r\}}$	Weighted Conditional Value at Risk at tolerance set $\{\gamma_1, ..., \gamma_r\}$