

**SOME NEW MODEL REDUCTION TECHNIQUES
FOR MULTIVARIABLE SYSTEMS**

By

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Dedicated to My
Brothers & Sisters

C E R T I F I C A T E

This is to certify that the thesis entitled,
'SOME NEW MODEL REDUCTION TECHNIQUES FOR MULTIVARIABLE
SYSTEMS' being submitted by B.Bandyopadhyay for the award
of the Degree of Doctor of Philosophy of the Indian Institute
of Technology, New Delhi, is a bonafide record of research
work carried out by him under my guidance and supervision.

The candidate has fulfilled the requirements of all
the regulations relating to the degree. The results obtained
in the thesis have not been submitted to any other University
or Institute for the award of a degree or diploma.

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A B S T R A C T

Applications of control techniques to large scale systems lead to mathematical system descriptions which are complex and of high dimension and thereby cause difficulties in the analysis and the design of controllers. A large number of simplification procedures are available for deriving reduced dimensional models of linear systems both in time as well as in frequency domain. Some methods of reduction are based on retention of dominant eigenvalues of the system. But these methods have certain limitations and disadvantages. Another important approach of model reduction is based on a matching of a number of time moments of the model and system. These methods are in frequency domain and have many useful features such as computational simplicity, the fitting of time moments etc. However, these methods may lead to unstable reduced model even though the original system is stable. This drawback is removed in the technique of Modal-Pade and Routh approximation procedure which are partial Pade in nature.

This thesis contains certain new model order reduction techniques for SISO and MIMO systems. Application of the proposed reduced models for controller design are also included. For employing many of control theory results to actual system analysis, time domain reduced order models prove to be very

useful. In this thesis Pade approximation and Modal-Pade methods for model simplification are extended to time domain both for single input single output (SISO) and Multivariable systems (MIMO).

The Routh approximation method uses the concept of Routh stability algorithm and an impulse response energy algorithm. A simplified state space version of the same for SISO system was developed by Rao et al. The Routh approximation for SISO system has many desirable features such as model stability, minimal computation, recursive nature of calculations for reduced models of various order etc. However, in many situations Routh models show unsatisfactory response behaviour in the initial part of the response. In this thesis it is shown that there is some amount of arbitrariness in the conventional Routh approximation procedure of Rao et al. and can be suitably utilised to obtain Modified Routh approximant. The superiority of the proposed Modified Routh approximant over the conventional and existing Improved and optimal Routh approximants is also discussed.

In the past attempts to extend the Routh approximation method for multivariable system did not prove very successful. In this thesis a method is presented for Routh approximation to multivariable system. Routh approximation methods for systems which are not controllable by a single input (being jointly controllable by all the inputs) as well as for systems

which have some uncontrollable modes are also proposed. A criterion for the choice of model order for multivariable system is also included.

Model reduction techniques based on error minimisation are examined. In the work reported in this thesis a simpler and more general frequency domain method of reduction by error minimisation is presented. The method is extended to multi-variable systems.

Applications of the SISO and MIMO Routh approximant reduced order models have been investigated for the purpose of designing suboptimal state and output feedback controllers. For state feedback suboptimal controllers stability condition for the resultant feedback system is also investigated.

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