ANALYSIS OF THE DYNAMIC RESPONSE OF LARGE ORTHOTROPIC ELASTIC PLATES TO TRANSVERSE IMPACT AND ITS APPLICATION TO FIBRE–REINFORCED PLATES

by

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CERTIFICATE

This is to certify that the thesis entitled "ANALYSIS OF THE DYNAMIC RESPONSE OF LARGE ORTHOTROPIC ELASTIC PLATES TO TRANSVERSE IMPACT AND ITS APPLICATION TO FIBRE-REINFORCED PLATES" being submitted by Mr. MOHAMMAD REZA KHALILI to the Indian Institute of Technology, Delhi for the award of the Degree of DOCTOR of PHILOSOPHY is a record of bonafide research work carried out by him under my supervision and guidance. The thesis work, in my opinion, has reached the standard fulfilling the requirements for the award of the degree of Doctor of Philosophy.

The research report and the results presented in this thesis have not been submitted in part or full to any other University or Institute for the award of any degree or diploma.

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ABSTRACT

With the increasing use of fibre-reinforced composite materials for primary structures, particularly in applications where weight saving is an important criterion, it is imperative to predict the response of such structures to impact loads which may be due to a falling weight or a projectile hit, etc. The response of fibre-reinforced composite structures to static or quasi-static loads is well understood and documented. Similarly, their behaviour under free as well as forced vibrations has been investigated in detail. However, the problem of transverse impact on a beam or plate of fibre-reinforced composite materials has received little attention.

For maximum efficiency of reinforcement, these materials are used in the form of aligned fibre beams, plates or shells. Such an arrangement is generally referred to as a lamina and it obviously exhibits orthotropic elastic behaviour. However, in more practical situations, particularly when a multi-axial stress field is expected in a structure, an assembly of laminae with different fibre orientations (referred to as a laminate) is used. In either of the two forms, the stress-strain relationship for the material is generally linear almost upto failure in case of a lamina and upto first ply failure in case of a laminate.

The present work deals with the dynamic response under

transverse impact of a large plate reinforced with aligned continuous fibres which can be viewed as a linear elastic orthotropic lamina. The behaviour of a laminate under certain restrictions of symmetry etc. can also be described by similar equations as for a lamina.

The impact load considered in this analysis need not be concentrated at a point, as has been assumed in most of the existing literature. Instead, the impact load distribution is based on Sveklo's theory of contact between anisotropic bodies which is a modification of the classical Hertz's theory for isotropic bodies. Accordingly, the force is considered to be distributed in a parabolic manner over an elliptical area whose size depends on the magnitude of the impact load and the elastic properties and geometry of the impacter and the plate.

The method of analysis adopted for the solution of the equation of motion uses Fourier transforms in contrast to the method involving series of eigenfunctions as used by most of the authors. The former yields a closed form solution in terms of integrals which can be evaluated numerically. An important feature of this analysis is the determination of the transform of the contact force distribution. A somewhat involved manipulation of the integrals finally yields a very simple expression for force transform and the special cases of a point load and a line distribution of load are noted.

The solutions are general enough to permit the determination of deflection and bending curvatures at an arbitrary point of the plate and not just at the centre of impact. Furthermore, the cases of an isotropic plate with a point load or a load distributed parabolically over a circular area are also discussed. In certain special cases, exact evaluation of the integrals is possible. In particular, for an isotropic plate subjected to a concentrated impact load, Boussinesq's classical solution is obtained. Similarly for an aligned fibre-reinforced plate under a concentrated load, the exact solution is possible for deflection and bending curvatures at the point of impact. These solutions had earlier been obtained by Frischbier using a different approach. Numerical solutions for plate deflection in those cases where exact solution is not possible are given. For this, two force histories have been chosen, viz. a rectangular pulse and a half-sine pulse. Deflection is obtained as a function of position and time.

A detailed analysis has been undertaken to determine the effect of the size of contact area on the deflection of the plate. It has been shown that upto an ellipse of semi-minor axis of approximately 1 mm for the plates chosen (aluminium for the isotropic case and graphite fibre-reinforced epoxy for the orthotropic case) the solution for the concentrated load provides a very good approximation, but for larger areas of contact there is a discernible effect. However, the effect of eccentricity of the ellipse is negligible within the limits considered.

In actual practice, the loading is generally caused by the impact of foreign object on the plate and the history of the impact force is itself unknown. Therefore, determination of impact force history is considered through relevant equations. The interaction between the plate and the impacting body is introduced through Hertz-Sveklo contact law. The effect of the size of the contact zone is neglected to begin with. Consequently, the deflection at the point of impact due to a concentrated load is given by a simple relation for isotropic as well as anisotropic plate, viz. the deflection is directly proportional to the impulse imparted to the plate. The constant of proportionality (plate parameter) depends only on the elastic constants, density and thickness of the plate. The non-dimensionalized integral equation for impact force also depends on a single parameter called as impact parameter, which is a function of plate parameter, mass and velocity of the impacter. This equation is solved by a simple algorithm. Curves are presented for the force history for different values of impact parameter. In order to consider the influence of the size of the contact zone, an iterative procedure has been recommended. After having determined the impact force history, it is now possible to determine the bending curvatures and hence the bending stresses and strains. Some exact solutions for few cases and their comparison with the existing literature were obtained. As has been pointed out by Sneddon for isotropic plates, there exists a similar singularity in the expressions for bending curvatures for orthotropic plates subjected to concentrated impact load. Schwieger has attempted to eliminate this singularity in a semi-empirical manner, but the present analysis uses an approach consistent with the theory of contact between two bodies.

In addition, the effect of the size and eccentricity of the contact ellipse on the bending curvatures has been investigated both for isotropic and orthotropic plates. Whereas the eccentricity has almost no effect, the size of the contact ellipse can have a discernible influence on curvatures when it exceeds a particular limit (a = .1 mm). A comparison is also carried out with Schwieger's semi-empirical procedure for the case of an isotropic plate. Results have been plotted for bending curvatures in various cases.

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