

ANALOG MODELS IN OPERATIONS RESEARCH

Thesis submitted to the Indian Institute of Technology, Delhi
for the award of the Degree of
DOCTOR OF PHILOSOPHY

by

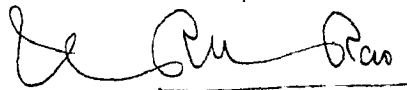
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CERTIFICATE

This is to certify that the thesis entitled ' ANALOG MODELS IN OPERATIONS RESEARCH ' by Shyam Maitra, has been prepared under my supervision, in conformity with the rules and regulations of the Indian Institute of Technology, Delhi. I further certify that the thesis has attained the standard required for the award of the degree of Doctor of Philosophy in Mechanical Engineering, of the Institute. The research report and results presented in the thesis have not been submitted elsewhere for the award of any other degree.



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SYNOPSIS

Several attempts have been made in the past to construct analog models to represent optimisation problems in Operations Research, in the areas of Linear Programming and Transportation. Of these, a mechanical analog model, originally proposed by Thomson for solving sets of linear equations, has been adapted by various researchers to represent optimisation problems, involving linear constraints. The physical models, constructed on these lines, suffer from several practical limitations, resulting in non-idealistic behaviour.

However, these mechanical models have the merit that they are particularly simple to visualise, providing an insight into the process of optimisation. The Kuhn-Tucker conditions which have to be satisfied by all optimal solutions, do not have to be artificially enforced. Instead, they follow from the requirement of equilibrium of forces on the system. The process by which the analog model proceeds towards equilibrium, suggests an algorithm based on the idealised behaviour of the system. This thesis examines mechanical analog models representing (a) Linear Programming (b) Transportation (c) Quadratic Programming problems. The approach, however, is general and can be extended to other optimisation problems as well.

The mechanical analog model, for the three types of problems considered, are only different versions of the basic Thomson model. The constructional features may have to be modified to suit the particular case, but the general procedure by which the models are brought to equilibrium, remains the same. Essentially, the model consists of an inter-connected system of levers and strings, such that the displacements of the levers are subject to certain restrictions (constraints). Weights of varying magnitude are suspended from the levers, so that the total energy dissipated by the system, corresponds to the objective function to be optimised. As all free systems, tend to minimise their energy (maximise the energy dissipated), the displacements of the levers, under equilibrium conditions provide the optimal solution.

Chapter (1) of this thesis describes the simulation procedure. The same basic procedure is followed in every case. Initially all the levers in the analog model are imagined to be at zero displacement, and are imagined to be subjected to certain unbalanced moments, which require external restrictions to keep the system in equilibrium. In the simulation procedure, these external

restrictions are relaxed one by one, in such a way as to maximise the energy dissipated. Each time one of these external restrictions is relaxed, the new displacements of the levers, and the new unbalanced moments have to be computed.

In chapter (2), the different versions of the Thomson model are described. The modifications are required in order to make the model applicable to the different types of optimisation problems considered.

In the third chapter, the simulation procedure is applied to the analog model for Linear Programs. It is shown that the simulation procedure provides a mechanical interpretation of the Simplex technique.

The fourth chapter deals with the simulation of the analog model for generalised Transportation problems.

In the fifth chapter the Quadratic Programming case is discussed. In this chapter, attention is confined to a model, which bears a resemblance to the Beale's algorithm.

Chapter (6) deals with the second form of the analog model for Quadratic Programming problems. The points of similarity with the Wolfe's algorithm are described.

In the concluding chapter, the process by which the simulation procedure can be extended to other optimisation problems, is discussed. Particular attention has been paid to optimising a logarithmic function subject to linear constraints. This might prove to be of use in Geometric Programming.

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