

A THEORETICAL STUDY ON THE EFFECT OF ANDREEV REFLECTION ON TRANSPORT PROPERTIES OF GRAPHENE- SUPERCONDUCTOR HETEROJUNCTIONS

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REFLECTION ON TRANSPORT PROPERTIES OF
GRAPHENE-SUPERCONDUCTOR HETEROJUNCTIONS

by

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CERTIFICATE

This is to certify that the thesis entitled, "**A theoretical study on the effect of Andreev reflection on transport properties of Graphene – Superconductor heterojunctions**", submitted by **Mr. Shahrukh Salim**, to the Department of Physics, Indian Institute of Technology Delhi, for the award of degree of **Doctor of Philosophy**, is a record of bonafide research work carried out by him under our supervision. He has fulfilled the requirements for the submission of the thesis, which to the best of our knowledge has reached the required standard. The results obtained in this thesis have not been submitted in part or full to any other University or Institute for the award of any degree or diploma.

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Dedicated to my loving family...

*In life, it's not where you go,
it's who you travel with.*

— Charles M. Schulz

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Abstract

In light of recent experimental advances, we explore the charge transfer process across a monolayer Graphene - Superconductor interface in graphene-based Josephson junctions. Our methodology is based on the transfer matrix method which is used to obtain the analytical expressions for the Andreev bound states (ABS) including the effects of retro Andreev reflection (RAR) and specular Andreev reflection (SAR) in different parameter regimes. We analytically derive the expressions for calculating the Josephson current and the conductance of the junctions using the ABS spectrum and compare them with some of the available experimental data for different values of the bias voltages and junction lengths. We find a good agreement between the theory and the experimental results and observe a scaling behaviour of the current and the conductance with respect to the junction length. We also discuss the role of SAR and RAR in determining the magnitude of the current and conductance in various parameter ranges. We extend these calculations to the SGSGS Josephson junction arrays and study the effect of the interference at different SG interfaces on the transport properties of the junction array, taking into account the retro and specular aspects of the Andreev reflection processes.

We also investigate the electrical properties of a Graphene - Superconductor - Graphene (GSG) hybrid structure, where a superconducting region is induced in the middle of a graphene sheet, in contrast to the widely studied electrical properties across a Superconductor-Graphene-Superconductor (SGS) type of Josephson junction. Using a method based on the transfer matrix, we mainly examine the Goos-Hänchen shift for the electrons and the holes at the GS interface in such a structure due to normal and Andreev reflection. Furthermore, we calculate the normalised differential conductance as a function of bias voltage that describes the transport through such junctions and highlight how Andreev and normal reflection affect them.

शोध सार

हाल ही के प्रायोगिक विकास को देखते हुए हमने एक मोनोलेयर ग्राफीन-अतिचालक अंतरफलक में आवेश अंतरण की प्रक्रिया का अध्ययन किया है। ये मोनोलेयर ग्राफीन-अतिचालक (Graphene-Superconductor interface) अंतरफलक जोसेफसन जंक्शन पर आधारित है। हमारी कार्यप्रणाली ट्रांसफर मैट्रिक्स पद्धति पर आधारित है। इस पद्धति के द्वारा हमने एण्डीव परिवर्द्ध अवस्थाओं (ABS) की विश्लेषणात्मक अभिव्यक्तियाँ निकाली है। ये विश्लेषणात्मक अभिव्यक्तियाँ हमने रेट्रो एण्डीव रिफ्लेक्शन (RAR) और स्पेक्युलर एण्डीव रिफ्लेक्शन (SAR), दोनों ही स्थितियों में निकाली है। इसके साथ ही हमने जोसेफसन प्रवाह और चालकता की भी विश्लेषणात्मक अभिव्यक्तियाँ निकाली है। इन अभिव्यक्तियों को निकालने के लिए हमने एण्डीव परिवर्द्ध अवस्थाओं का उपयोग किया है। हमने इन परिणामों की वर्तमान में मौजूदा परिणामों से तुलना की है। हमने पूर्वाग्रह वोल्टेज और अलग-अलग जंक्शन लम्बाई के लिए इन परिणामों की तुलना की है। इस तुलना में हमने ये पाया की हमारी सैद्धांतिक पद्धति पहले से मौजूद पद्धतियों से गुणात्मक रूप से मेल खाती है। हमने ये भी पाया की जोसेफसन प्रवाह और चालकता जंक्शन की लम्बाई के साथ स्केलिंग व्यवहार दिखाती है। जोसेफसन प्रवाह और चालकता का अलग अलग मापदंडों के साथ निर्धारण करने के लिए हमने रेट्रो एण्डीव रिफ्लेक्शन और स्पेक्युलर एण्डीव रिफ्लेक्शन की भूमिकाओं पर भी प्रकाश डाला है। हमने इस विश्लेषणात्मक पद्धति का उपयोग करके एसजीएसजीएस (SGSGS) जोसेफसन जंक्शन सरणी में अलग-अलग एस-जी (SG) अंतरफलक पर होने वाली अंतःक्रिया का अध्ययन किया है। हमने इसके साथ ही इन अंतःक्रियाओं के होने की वजह से परिवहन गुण पर पड़ने वाले प्रभाव का भी अध्ययन किया है। इन प्रक्रियाओं में हमने रेट्रो एण्डीव रिफ्लेक्शन और स्पेक्युलर एण्डीव रिफ्लेक्शन की भूमिकाओं को समझने का प्रयास किया है।

हमने जीएसजी (GSG) संकर जंक्शन के विद्युत् गुणों का भी अध्ययन किया है। यहां पर अतिचालक क्षेत्र ग्राफीन शीट के मध्य में बनाया गया है। यह स्थिति एसजीएस (SGS) संकर जंक्शन के बिलकुल विपरीत है, जो की एक जोसेफसन जंक्शन है। ट्रांसफर मैट्रिक्स पद्धति का प्रयोग करते हुए जीएस जंक्शन में इलेक्ट्रान और होल में जीएच (GH) गूस-हैनचेन शिफ्ट का अध्ययन किया है। ये अध्ययन हमने एण्डीव और सामान्य दोनों तरह की प्रक्रियाओं के लिए किया है। इसके साथ ही हमने जीएसजी जंक्शन में विद्युत् चालकता की पूर्वाग्रह वोल्टेज के फंक्शन के तौर पर परिगणना की है। हमने इन जंक्शनों में विद्युत् चालकता पे एण्डीव और सामान्य प्रक्रियाओं की वजह से पड़ने वाले प्रभाव का भी अध्ययन किया है।

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