

**TRANSIENT DYNAMIC FINITE ELEMENT ANALYSIS
OF PLATE AND SHELL STRUCTURES
WITH FINITE ROTATION**

by

A. LAZAR

DEPARTMENT OF APPLIED MECHANICS

submitted

**in fulfilment of the requirements
for the degree of
DOCTOR OF PHILOSOPHY**

to the



INDIAN INSTITUTE OF TECHNOLOGY, DELHI

DECEMBER, 1996

CERTIFICATE


This is to certify that the thesis entitled 'Transient Dynamic Finite Element Analysis of Plate and Shell Structures with Finite Rotations' being submitted by Mr. A.Lazar to the Indian Institute of Technology, Delhi for the award of the Degree of Philosophy in Applied Mechanics is a record of bonafide research work carried by him under our supervision and guidance. The thesis work, in my opinion, has reached the requisite standard fulfilling the requirement for the Doctor of Philosophy Degree.

The results contained in this thesis have not been submitted in part or in full, to any other University or Institute for the award of any degree or diploma.



(Dr. Suhail Ahmad)

*Associate Professor,
Deptt. of Applied Mechanics
Indian Instt. of Technology
Delhi - 110016*



(Prof. C.V.Ramakrishnan)

*Professor,
Deptt. of Applied Mechanics
Indian Instt. of Technology
Delhi - 110016*

ACKNOWLEDGEMENTS

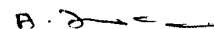
I would like to express my deep sense of gratitude to my supervisors Prof. C.V.Ramakrishnan and Dr. Suhail Ahmad for their meticulous guidance, kind help, moral support and constant encouragement throughout this research work.

I am thankful to the Holtec Consulting Ltd., New Delhi for their help and timely assistance.

Thanks are also due to Prof. G.S.Shekon, Head, Dept. of Applied Mechanics, Prof. P.K.Sen, Chairman DRC for their kind help and support. I am also indebted to all the faculty members of Applied Mechanics Department who helped me at various stages of this research work.

The prompt help and co-operation rendered by Mr. Sriram Hegde, System Administrator Computational Laboratory, is highly acknowledged. I am also thankful to my co-researchers Dr.A.C.Paul, Mr.G.V.V.Ravi Kumar, Mr.Anajn Dutta, Mr.R.Marimuthu, Dr.R.Velumurugan, Miss.Sheela Sathianeson, Miss.Aleyamma, Mr.Rama Subramaniam without whose help, this work could not have progressed smoothly. Thanks are also due to Miss Richa Nigam, Mr.M.S.Raghunath, Mr. V.S.Rawat JTA , Mr. Kailash Chand STA, Computational Laboratory for their cooperation.

Finally, I would like to express my gratitude to wife Leoni and sons Nivas Daniel and Naveen D'souza for their cooperation, understanding, moral support and encouragement that made it possible for me to complete this research work.



(A.Lazar)

ABSTRACT

In problems related to the dynamics of plates and shells, one frequently encounters situations involving large displacement and large rotations giving rise to highly nonlinear problems. In the first stage, an one dimensional counterpart of governing dynamic finite element equation of motions with geometric nonlinearity, namely Duffing's equation, has been studied using Incremental procedures Pseudo force, Tangent force and Pseudo-tangent force method. The Duffing's equation has been solved using different solution schemes such as direct integration and incremental procedures and a critical comparative study of responses of different solution schemes has been presented.

The nonlinear dynamic finite element equations of motion has been discussed and various finite element matrices connected with the dynamic equation of motion have been derived in a systematic manner. Formulation of dynamic element equations of motion with large displacement and small rotation effects using different methods of incremental modal techniques and direct integration method have been discussed. Time dependant response have been obtained using direct integration and Incremental modal superposition method and a detailed comparison of the solutions has been made.

The dynamic finite element equations of motion with large displacement and finite rotation effects have been derived and the various matrices related to the system have been presented. Detailed response studies have been conducted for various practical problems and a comparative study has been presented to discuss the role finite rotation effects.

The dynamic finite element equations of motion with large rotation and small rotation effects applicable to composite materials have been derived using a layered approach. A comparative study of static and dynamic response with the effects of large rotation vis-a-vis small rotation formulation for practical problems has been presented for several practical problems. A detailed discussion on these results has been presented.

Detailed discussions of results and suggestions for future research are discussed.

TABLE OF CONTENTS

	Page
Certificate	i
Acknowledgements	ii
Table of Contents	iii
Abstract	ix
List of Figures	xi
List of Tables	xv
Nomenclature	xvi

CHAPTER 1

1.0 INTRODUCTION AND REVIEW OF LITERATURE	1-24
1.1 Introduction	1
1.2 Overview Computation Shell Element Formulation and Analysis	2
1.2.1 Classical shell theory formulation	2
1.2.2 Classical shell theory based on Cosserat surface approach	5
1.3 Concepts of Lagrangian and Eulerian Formulation	5
1.3.1 Definition of formulation	5
1.3.1.1 Total lagrangian formualtion	6
1.3.1.2 Updated lagrangian formualtion	7
1.3.1.3 Co-rotational formulation	8
1.4 Comments Regarding Formulation	9
1.5 Review of Procedures for Geometrically Nonlinear Finite Element Shell Analysis	12
1.5.1 Ramm's method	15
1.5.2 Oliver and Onate's method	16
1.5.3 Parisch and Surana's method	17
1.5.4 Bathe and Bolourchi's method	17
1.5.5 Finite rotation method	18
1.5.6 Mixed finite rotation method	19
1.6 Scope of Present Investigation	20
1.7 Objective of the Present Work	23
1.8 Organization of the Thesis	23

CHAPTER 2

2.0 ALGORITHM FOR THE SOLUTION OF NONLINEAR DYNAMIC PROBLEMS	25-44
2.1 Introduction	25
2.2 Different Solution Strategies	25
2.2.1 Direct integration	25
2.2.2 Newmark- β method	27
2.2.3 Incremental modal technique	27

2.3 One Dimensional Incremental Algorithm	29
2.3.1 Formulation and solution of linear Duffing's equation in incremental form	30
2.3.2 Formulation and solution of nonlinear Duffing's equation in incremental form	31
2.3.2.1 <i>Pseudo force method</i>	31
2.3.2.2 <i>Tangent force method</i>	32
2.3.2.3 <i>Pseudo tangent force method</i>	33
2.3.3 Newmark- β method	34
2.4 Comparative Study of One Dimensional Incremental Algorithm with Newmark- β Method	36
2.4.1 Harmonic excitation-nonlinear	36
2.4.2 Suddenly applied load-nonlinear	38
2.4.3 Harmonic excitation-linear	40
2.4.4 Suddenly applied load case-linear	42
2.5 Comparison of Different Formulation of Incremental Method	43
2.6 Closure	44

CHAPTER 3

3.0 FINITE ELEMENT EQUATIONS FOR SHELL DYNAMICS AND THEIR SOLUTION	55-101
3.1 Introduction	55
3.2 Derivation of Discretized System Equation of Motion for Shell Dynamic Analysis	55
3.3 Derivation, Definition of Various Finite Element parameters	58
3.3.1 Shell elements in general	59
3.3.2 Degenerate isoparametric shell elements	
3.3.3 Coordinate system	59
3.3.3.1 <i>Global coordinate system</i>	61
3.3.3.2 <i>Nodal coordinate system</i>	61
3.3.3.3 <i>Curvilinear coordinate system</i>	62
3.3.3.4 <i>Local coordinate system</i>	64
3.3.4 Element geometry	64
3.3.5 Numerical integration	65
3.3.6 Element library	67
3.3.6.1 <i>Serendipity element</i>	68
General	68
Shape function	68
Displacement field	69
3.3.6.2 <i>Lagrangian element</i>	71

<i>General</i>	71
<i>Shape function</i>	71
<i>Programming formulation</i>	73
<i>Displacement field</i>	73
3.3.6.3 <i>Heterosis element</i>	74
3.3.7 Definition of strains	76
3.3.8 Strain displacement matrix	77
3.3.8.1 <i>Linear strain displacement matrix B_0</i>	77
3.3.8.2 <i>Nonlinear strain displacement matrix B_L</i>	80
3.3.9 Material property matrix D	82
3.3.10 Definition of stresses	82
3.3.11 Element properties	83
3.3.11.1 <i>Stiffness matrix</i>	83
<i>Linear stiffness matrix</i>	83
<i>Nonlinear stiffness matrix</i>	83
3.3.11.2 <i>Mass matrix</i>	86
3.4 Finite Element Equation of Motion for Discretized System for Shell Dynamics with Large Displacement, Small Rotation and Small Strain	86
3.5 Numerical Integration of Finite Element Equation of Motion	87
3.5.1 Incremental modal techniques with large displacement	87
3.5.1.1 <i>Pseudo force method</i>	87
3.5.1.2 <i>Tangent force method</i>	90
3.5.1.3 <i>Pseudo tangent force method</i>	91
3.5.2 Direct integration using Newmark β method	91
3.6 Comparative study of Structural response using the modal superposition and direct integration method using Newmark- β method with large displacement formulation	93
3.6.1 Methods of solution	93
3.6.2 Example	94
3.6.3 Linear response of cantilever plate	94
3.6.4 Nonlinear response of cantilever plate	96
3.7 Closure	98

CHAPTER 4

4.0 COMPARATIVE STUDY OF LARGE DISPLACEMENT AND FINITE ROTATION FINITE ELEMENT SOLUTION	102-173
4.1 Introduction	102
4.2 Large Displacement, Large Rotation and	

Small Strain Formulation	102
4.2.1 Derivation of nonlinear function	103
4.2.2 Derivation of B_0 matrix	107
4.2.3 Derivation of $\{\bar{\theta}\}$ in Local Axes System	112
4.2.4 Derivation of nonlinear strain displacement matrix	114
4.2.5 Derivation of stiffness matrices	115
4.2.5.1 <i>Tangent stiffness matrix K_T</i>	115
4.2.5.2 <i>Matrix K_{o1}</i>	115
4.2.5.3 <i>Matrix K_{o2}</i>	116
4.2.5.4 <i>Matrix K_{o3}</i>	120
4.3 Large Displacement Large Rotation Small Strain Formulation Validation- Static Analysis	124
4.4 Static Response of Cantilever Plate subjected to Moment at Free End-Comparative Study of Results of Small and Large Rotation Analysis	131
4.5 Static Analysis of Cylindrical and Spherical Shell	132
4.5.1 Static response of clamped shell subjected to pressure load	132
4.5.2 Static response of cylindrical shell with two longitudinal straight opposite edges hinged and two circular edges free subjected to central point load	134
4.5.3 Static response of clamped hemi-spherical rubber shell subjected to point load at crown	137
4.6 Discretized Finite element system equations of motion for shell dynamics with large displacement large rotation and small strain	141
4.7 The Comparative study of Dynamic response of cantilever plate subjected to a suddenly applied moment using different formulation	143
4.7.1 General	145
4.7.2 Linear analysis	145
4.7.3 Large displacement small rotation	145
4.7.4 Large displacement and large rotation	148
4.8 The Comparative Study of Dynamic Response of Cylindrical Shell subjected to Suddenly Applied Pressure Load	157
4.8.1 Dynamic response of all four side clamped cylindrical shell subjected to suddenly applied	

pressure load	157
4.8.1.1 <i>General</i>	158
4.8.1.2 <i>Linear analysis result</i>	158
4.8.1.3 <i>Large displacement small rotation</i>	159
4.8.1.4 <i>Large displacement, large rotation small strain</i>	159
4.8.2 Dynamic response of cylindrical with two longitudinal opposite straight edges fixed and two opposite circular edges free subjected to central point load	164
4.8.3 Dynamic response of cylindrical with two longitudinal straight opposite edges hinged and two opposite circular edges free subjected to central point load	165
4.9 The Comparative Study of Clamped Hemi Spherical Rubber Shell subjected to Suddenly Applied Point Load at Crown	168
4.9.1 General	170
4.9.2 Linear analysis	171
4.9.3 Large displacement small rotation	171
4.9.4 Large displacement and large rotation	171
4.10 Closure	173
CHAPTER 5	
5.0 FINITE ELEMENT EQUATIONS FOR COMPOSITE SHELL DYNAMICS AND THEIR SOLUTION WITH FINITE ROTATION	174-204
5.1 Introduction	174
5.2 Constitutive Laws for Composite Materials	175
5.2.1 Stress-strain relation	175
5.2.2 Elasticity matrix	176
5.2.1.1 Elastic material property matrix along fibre direction C	176
5.2.1.2 Elastic material property matrix composite materials D	177
5.3 Layered Approach	178
5.3.1 General technique	178
5.3.2 Stiffness matrix	180
5.3.2.1 Large displacement small rotation and small strain	180
5.3.2.2 Large displacement, large rotation and small strain	180
5.4 Finite Element Formulation of Dynamic Equation of Motion in Total Lagrangian Coordinates for Layered Composite Shell	183

5.5 Linear Analysis of Composite Cantilever Plate under Static Loads	183
5.6 Static Analysis of Composite Clamped Cylindrical Shell under Pressure Load	185
5.7 Dynamic Analysis of Multilayered Composite Cantilever Plate	187
5.7.1 General	189
5.7.2 Linear analysis result	191
5.7.3 Large displacement small rotation	191
5.7.4 Large displacement, large rotation and small strain	192
5.8 Dynamic Response of Cylindrical with Two Longitudinal Straight Opposite Edges Hinged and Two Opposite Circular Edges Free subjected to Central Point Load	192
5.8.1 General	197
5.8.2 Linear	197
5.8.3 Large displacement small rotation	197
5.8.4 Large displacement, large rotation and small rotation	198
5.9 The Comparative Study of Clamped Hemi Spherical Composite Rubber Shell subjected to Suddenly Applied Point Load at Crown	198
5.9.1 General	201
5.9.2 Linear analysis	202
5.9.3 Large displacement small rotation	202
5.9.4 Large displacement and large rotation	202
5.10 Closure	204
CHAPTER 6	
6.0 CONCLUSION AND SCOPE FOR FURTHER RESEARCH	205-207
6.1 Introduction	205
6.2 Analysis of Results and Summary of Conclusions	205
6.3 Significant Contributions of the Present Research	207
6.4 Suggestions for Further Research	207
REFERENCES	208