HARMONIC GENERATION AND SELF FOCUSING OF ELECTROMAGNETIC WAVES IN GASEOUS AND SEMICONDUCTOR PLASMA

by

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PREFACE

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The interaction of electromagnetic waves with gaseous and semiconductor plasmas has aroused the interest of a large number of theoretical as well as experimental investigators because of its numerous applications in many fields of science and technology. If the electric vector of the electromagnetic wave is weak, the current density in the plasma is directly proportional to the electric field. However, for moderately strong electric field the current density in the plasma becomes a nonlinear function of the electric vector. This occurs on account of the dependence of the complex conductivity complex permittivity of the medium on the electric vector of the propagating wave. A large number of interesting nonlinear phenomena in gaseous and semiconductor plasmas have been investigated in detail. The criterion for the nonlinear effects to be important is given in terms of an electric field E_p which is referred to as the plasma field (Ginzburg, 1970). Its value is given by

$$E_p = \left[3\, m\, k_o T\, \delta\, (\omega^2 + \nu^2)/e^2\right]^{1/2},$$

where

$-e$ is the electronic charge

m is the electronic mass (it is effective mass of the electron in the case of a semiconductor plasma)

ν is the collision frequency of electrons

T is the temperature of the plasma

k_{ω} is the Boltzmann constant

ω is the frequency of the wave field

and δ is the fraction of the energy transferred by the electron to a heavy particle in each collision.

If the applied electric field E is less than E_p it is referred to as weak; if $E > E_p$, the field is moderately strong and if $E >> E_p$, the field is said to be very strong. Phenomenologically, the nonlinear dependence of the effective dielectric constant of a plasma/semiconductor on the electric field can be understood as follows. The free carriers absorb energy from the field and in the steady state attain a temperature higher than the equilibrium value such that the rate of power absorption from the field becomes equal to the rate of power lost by the carriers by thermal conduction and by collisions with heavy particles. (Conwell, 1967; Ginzburg, 1970). Such carriers whose temperature is modified appreciably by the field are termed hot carriers and may give rise to the following important effects:

i) The mobility and hence the conductivity of carriers changes on account of the energy dependence of the carrier mass; the latter is important for nonparabolic semiconductors like indium antimonide (see for example Conwell, 1967).

ii) The local concentration of carriers may change on account of (a) nonuniform heating and consequent redistribution of carriers if the electric field is non-uniform, and (b) the breakdown of the plasma (see for example Sodha, Ghatak and Tripathi, 1974; Shkarofsky, Johnston and Bachynski, 1966).

These modifications in carrier parameters lead to the nonlinear relationship between the current density and the electric vector and hence to a field dependent conductivity or effective dielectric constant. A vast literature on the behaviour of hot carriers in semi-conductors (Conwell, 1967) and plasmas (Ginzburg, 1970) is available; such studies have found many applications in semiconductor devices and plasma diagonostics. In the present thesis we have investigated some interesting aspects of two well-known nonlinear phenomena viz.,

(i) Self focusing by nonuniform beams in semiconductors and plasmas.

(ii) Harmonic generation by uniform beams in non-parabolic semiconductor and by nonuniform beams in plasma; in the latter investigation self-focusing has been taken into account.

In the case of uniform beams the dominant nonlinearity in the effective dielectric constant of plasmas and semiconductors arises through the energy dependence of the relaxation time; the energy dependent carrier mass is an additional source of nonlinearity in nonparabolic semiconductors. In the case of nonuniform beams the dominant nonlinearity in the effective dielectric constant of plasmas and semiconductors having both types of carriers arises through the heating and redistribution of carriers. The redistribution of carriers is also caused by the ponderomotive force which is mainly important in collisionless plasmas. It is interesting to mention here that owing to the space charge effects the positive and negative charge carriers are redistributed through ambipolar diffusion. The nonlinearity arising through the energy dependence of carrier mass (in the case of nonparabolic semiconductors) is also important in causing the self focusing of the beam. The other types of nonlinearities (e.g. ponderomotive) are insignificant in steady state self focusing.

In what follows we give a brief account of self-focusing followed by that of harmonic generation.

SELF FOCUSING

The phenomenon of self focusing which arises primarily due to the dependence of the complex dielectric constant on the intensity of the propagating wave, is an important problem in nonlinear optics because it considerabl influences all other nonlinear phenomena e.g. harmonic generation and stimulated Raman scattering etc. This phenomenon has been extensively investigated in dielectrics, in recent years (Akhmanov, Sukhorukov and Khokhlov, 1968; Svelto, 1972 and Sodha, Ghatak and Tripathi, 1974).

The self focusing of laser beams in plasmas has recently attracted a great deal of attention on account of its applicability to controlled fusion using lasers. The instabilities of nonlinear origin in the plasma are enhanced by the enhancement of the intensity of the beam due to self focusing. The phenomenon of self focusing in plasmas is also relevant to the heating of the ionosphere by a ground based high frequency transmitter (Lelevier 1970; Meltz and Lelevier 1970, Perkins and Kaw 1971, Utlaut 1970) and should be of interest in plasma diagonostics.

The self focusing of laser beams in semiconductors has also been a field of sizable activity in recent years; the motivation for this work appears to be a better understanding of free carrier nonlinearities and the characterization of semiconductors.

The self focusing of electromagnetic waves in plasmas appears to be first discussed by Askaryan (1962) and

Talanov (1964). It was shown in their analyses that intense radiation of a laser beam can lead to a difference between the properties of the medium inside and outside the beam, which creates conditions suitable for self focusing of the beam, thereby eliminating the geometrical and diffraction divergences. Experimental investigations of self focusing effects began in 1965 after appearance of a fundamental paper by Chiao, Garmire and Townes (1964) on the self focusing of light beams in nonlinear dielectrics. An interesting conclusion of the investigation is that self focusing can occur only when the power of the light beam exceeds a threshold value, known as the critical power for self focusing.

It would be useful to visualize physically the phenomenon of self focusing in a medium where the refractive index is an increasing function of the intensity EE* of the beam (say $n = n_0 + n_2 E E^*$). We consider a plane uniform wavefront incident on a circular aperture (of radius a) in a nonlinear medium. The portion of the medium illuminated by the beam has a refractive index $(n = n_0 + n_2 E E^*)$ higher than that of the nonilluminated portion. Therefore, the secondary wavelets diverging at an angle θ from the wave normal suffer total internal reflection at the boundary of the fictitious cylinder of radius a when $\theta < \theta_c$, where $\theta_c = \cos^{-1}\left(\frac{n_0}{n}\right)$ corresponds to the critical angle. It is also known from the diffraction theory that a very large fraction of the power will be carried by rays making an angle less than θ_D with the axis, where

$\theta_D = 0.61\lambda_0/2an_0$,

and λ_0 is the wavelength of the radiation in free space. We may consider three obvious possibilities -

(i) When $\theta_D > \theta_c$ the beam will diverge due to predominance of diffraction effects.

(ii) When $\theta_D = \theta_c$ the beam should propagate in the self made waveguide. The corresponding power of the beam can easily be shown to be given by

$$P = P_{cr} = (1.22\lambda_0)^2 c/128 n_2 ,$$

where P_{cr} is the critical power of the beam and c is the velocity of light in vacuum.

(iii) When $\theta_D < \theta_c$ one expects the beam to focus.

The above speculations are supported by rigorous analyses. The author has investigated the phenomenon of self focusing with reference to two specific cases (i) non-degenerate sample of indium antimonide and (ii) strongly ionized plasma. These two cases have been considered separately to show the importance of the two mechanisms of energy loss viz. the collisional loss and the conduction loss. The analysis for a strongly ionized plasma has been extended to include the effect of a static magnetic field; the relative predominance of conduction and collisional losses is reversed by the application of appreciable magnetic field. In each case expressions for the field dependent dielectric constant have been derived and the propagation and focusing of an electromagnetic wave (having Gaussian intensity distribution along its wavefront) in different cases have been studied by solving the wave equation for different dependences of the complex dielectric constant on the electric vector. The treatment is based on the technique developed by Akhmanov, Sukhorukov and Khokhlov (1968).

HARMONIC GENERATION

Since electric field of an electromagnetic wave is alternating, the nonlinear effects in the plasma lead to the generation of odd harmonic components in the current density (Rosen, 1961). Baird and Coleman (1961) and Murphy (1965), on the basis of elementary approach have pointed out that in addition to the odd harmonic components in the current density, even harmonic components in the current density are also generated when a plasma is subjected to a dc electric field and an alternating electric field simultaneously. Chiyoda (1965) and Sodha and Kaw (1966) suggested that even harmonic components in the current density can also be generated if the alternating electric field is applied to an inhomogeneous plasma having gradients of temperature and electron density. An excellent review of harmonic generation is given by Sodha and Kaw (1969). An important consequence of the generation of these higher order components in the current density is the generation of electromagnetic waves at these harmonic frequencies. Physically, the phenomenon of harmonic generation can be understood as follows.

The power absorbed by the electrons from the wave of frequency ω has a dc component and a time varying component oscillating with a frequency 2ω; the latter leads to an oscillatory component of electron temperature. This oscillating component of electron temperature causes the modulation of the effective collision frequency. Since the conductivity is a function of the collision frequency, the conductivity also has a component oscillating with frequency 2ω. This nonlinear part of the conductivity will interact with the primary wave to give third harmonic component in the current density. It is obvious that the second harmonic component in the current density will also be generated if in addition to the alternating electric field a dc electric field is also applied. The dependence of effective mass on carrier temperature (energy) also leads to the same qualitative conclusions.

In this thesis we have investigated the problem of microwave harmonic generation in a degenerate non-parabolic semiconductor with special reference to n-type indium antimonide in the presence of a dc electric field, an alternating electric field and a static magnetic field using rigorous kinetic approach. The usual procedure of cartesian tensor expansion of the electron velocity distribution function has been employed.

The analysis of harmonic generation in plasmas has been restricted to the case of nonfocusing waves; the theory predicts that the intensity of the n^{th} harmonic is proportional to the n^{th} power of the intensity of the fundamental wave. The phenomenon of self focusing (arising on account of the intensity dependence of the refractive index) leads to an increase in the average value of the n^{th} power of the intensity of the fundamental beam and hence is expected to lead to higher harmonic outputs. Therefore, it is necessary to include the phenomenon of self focusing in any realistic analysis of harmonic generation. We have explored this seemingly inevitable influence of self focusing on the generation of third harmonic in a collisional plasma by a Gaussian electromagnetic beam whose power exceeds the critical power for self-focusing. The total power of the third harmonic incident across a transverse cross-section has been evaluated for different values of the distance of propagation of the main beam in the plasma. The contents of the thesis have been divided into three parts.

Part I consists of three chapters and deals with the phenomenon of self focusing of electromagnetic waves in gaseous and semiconductor plasmas.

Part II consists of one chapter and deals with the phenomenon of harmonic generation in nonparabolic semiconductors.

Part III consists of one chapter and gives an account of the effect of self focusing on harmonic generation by a Gaussian electromagnetic wave in a plasma.

A chapterwise summary is given below:

Chapter-I : Self-focusing of Electromagnetic Beams in Indium Antimonide: Predominance of Nonlocal Effects

This chapter presents an investigation of self focusing of Gaussian electromagnetic beams in a non-degenerate, nonparabolic semiconductor like indium antimonide (having both +ve and -ve carriers). Our analysis is restricted to low temperature range so that the ionized impurity scattering is the dominant scattering mechanism for momentum transfer. The energy transfer in this temperature range is attributed to optical phonon scattering. The nonlinearities responsible for self focusing are caused by (i) energy dependent relaxation time (ii) nonuniform heating and consequent redistribution of carriers along the wavefront and (iii) nonparabolicity of conduction band. The relaxation time nonlinearity is not significant in causing self focusing at high frequencies ($\omega >> \nu$) as shown by Sodha, Ghatak and Tripathi (1974). We have given an account of redistribution mechanism for nonlinearity when the relative concentration of electrons and holes is arbitrary; a comparison of the redistribution nonlinearity with the mass nonlinearity (introduced due to nonparabolicity of energy bands) has also been made for typical parameters. It is seen that the former is about one order of magnitude higher than the latter at low temperatures.

Chapter-II : Self-focusing of a Laser Beam in a Strongly Ionized Plasma

In this chapter the author has studied the self focusing of a Gaussian laser beam in a strongly ionized plasma. The nonlinearity in the dielectric constant arises due to the nonuniform heating and consequent redistribution of the carriers. The excess energy gained by the hot electrons from the wave field is dissipated in collisions and in thermal conduction. In the case of a strongly ionized plasma the loss of energy due to thermal conduction is of much importance and dominates over the collisional loss in contrast to the case of a weakly ionized plasma where reverse is true.

Taking conduction loss as the effective mechanism for energy dissipation we have derived the expression for the nonlinear dielectric constant. This nonlinearity in the dielectric constant is effective in causing the self focusing of the beam. The results show that the beam propagates in a self made oscillatory waveguides, even when the nonlinear dielectric constant does not fall in the saturating range. In a typical case of a 10^{10} watt laser, the enhancement of axial intensity by a factor of 25 has been predicted in a feasible length of 0.6 cm.

Chapter-III : Self-focusing of Electromagnetic Beams in a Strongly Ionized Magnetoplasma

In this chapter the author has extended the treatment of chapter 3 to include the effect of a static magnetic field. The presence of a static magnetic field makes the plasma anisotropic and the dielectric constant becomes a tensor. The nonlinearity in the dielectric tensor of the plasma arises due to the heating and consequent redistribution of electrons along the wavefront in this case also. At low values of the magnetic field the main source of electron energy dissipation is thermal conduction; for large magnetic fields carrier mean free path becomes comparable to or greater than the Larmour radius of electrons and hence the collisional loss of energy becomes dominant. The self focusing of the two modes of propagation has been studied by considering the presence of only one mode at a time. The investigation reveals that the length of self focusing of the beam in the extraordinary as well as ordinary mode of propagation is reduced by the application of the magnetic field for low values of the magnetic field ($\omega_c \ll \omega$). For higher values of the magnetic field the self focusing of extraordinary mode is further enhanced while that of ordinary mode is reduced.

Chapter-IV : Effects of Magnetic Field on Microwave Harmonic Generation in n-Type Indium Antimonide

In this chapter the author has investigated the phenomenon of harmonic generation in a degenerate n-type indium antimonide slab when the latter is subjected to an external dc electric field, an alternating microwave field and a static magnetic field. The microwaves are assumed to be travelling along the direction of the static magnetic field. In this investigation only the free carrier nonlinearities have been considered. The nonlinearity arising due to the polarization of the background lattice is important only at optical frequencies and hence it is ignored in our analysis. The free carrier nonlinearities arise due to (i) energy dependence of the carrier relaxation time and (ii) nonparabolicity of conduction band. Since our treatment is restricted to relatively low temperatures, ionized impurity scattering has been considered to be the sole mechanism for scattering of carriers.

In this investigation the author has employed rigorous kinetic approach based on Boltzmann transfer equation. Using Boltzmann transfer equation in k-space for electrons in an n-type indium antimonide sample, the author has derived explicit expressions for the second and third harmonic components of the current density for both the extraordinary and ordinary modes of propagation. These expressions for the alternating components of the current density are substituted in the appropriate wave equation and the resulting nonlinear differential equations have been solved to obtain expressions for the amplitudes of second and third harmonic waves in the reflected and transmitted components of the electric vector. To appreciate the effect of the magnetic field on the magnitudes of the generated harmonics, some computed results have been presented. It is seen that there is an overall decrease in the amplitude of the second harmonic with static magnetic field. The amplitudes of the extraordinary mode of the third harmonic increases with the magnetic field while that of ordinary mode decreases. Helicon effects are seen to be more marked at high magnetic fields.

Chapter-V : Nonlinear Third Harmonic Generation in a Plasma by a Gaussian Electromagnetic Beam: Effect of Self-focusing

In this chapter the nonlinear third harmonic generation in a collisional plasma by a Gaussian electromagnetic beam has been investigated when the power of the beam exceeds the critical power for self-focusing. The third harmonic component of current density is generated on account of energy dependent collision frequency. The self focusing, on the other hand, is caused by the redistribution of charged particles along the wavefront. As a matter of fact the self focusing of the beam becomes significant at powers of the beam much less than those needed for appreciable harmonic output.

Using explicit expression of the third harmonic component of current density in the general wave equation, the author has derived the expression for the magnitude of the third harmonic field produced by a Gaussian beam taking self focusing into account. Two specific cases of nonlinearity have been considered in detail
(i) quadratic nonlinearity (ii) saturating nonlinearity.
In each case the power of the third harmonic incident across a transverse cross-section has been evaluated as

a function of the distance of propagation of the fundamental wave in the plasma. Numerical calculations have been carried out for typical parameters in both the cases to appreciate the role of self focusing in increasing the harmonic power output. The present investigation reveals that the self focusing of the main beam considerably enhances the harmonic power incident across a transverse cross-section; in a typical case the enhancement of harmonic power is as high as three orders of magnitude in the paraxial ray approximation.

The work reported in the thesis has resulted in the following publications:-

1. Magnetic Field Effects on Microwave Harmonic Generation in n-InSb, J.Phys.D., 4, 552 (1971).
2. Self-focusing of Electromagnetic Beams in Indium Antimonide: Predominance of Nonlocal Effects, Appl.Phys. (Germany) 2, 39 (1973).
3. Self-focusing of a Laser Beam in a Strongly Ionized Plasma, Opto Electronics, 5, 533 (1973).
4. Self-focusing of Electromagnetic Beams in a Strongly Ionized Magnetoplasma, J.Phys.D., 7, (1974).
5. Nonlinear Third Harmonic Generation in a Plasma by a Gaussian Electromagnetic Beam: Effect of Self-focusing (Communicated).

CONTENTS

PART - III