

**ANALYSIS OF SOME INDENTATION MODELS
RELATED TO INCIPIENT DEFORMATION OF BRITTLE & DUCTILE
WORKMATERIALS DURING ORTHOGONAL METAL CUTTING**

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**SUBMITTED
IN FULFILMENT OF THE REQUIREMENTS OF
THE DEGREE OF DOCTOR OF PHILOSOPHY**



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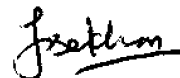
CERTIFICATE

This is to certify that the thesis entitled "Analysis of Some Indentation Models related to Incipient Deformation of Brittle and Ductile Workmaterials during Orthogonal Metal Cutting", being submitted by Mr. Jagdish Raj Anand to the Indian Institute of Technology, Delhi, for the award of Degree of Doctor of Philosophy is a record of bonafide research work carried out by him. He has worked under our guidance and supervision and has fulfilled the requirements for the submission of the thesis which, to our knowledge, has reached the requisite standard.

The results contained in this thesis have not been submitted in part or full, to any other University or Institute for the award of any Degree or Diploma.



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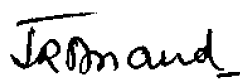


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Summary of Revisions Incorporated

The author is grateful to the examiners for their learned comments on the thesis. He has carefully gone through the comments and revised the original manuscript as follows:

- (i) The title has been changed from the earlier "Analysis of Incipient Deformation of Workmaterial under Orthogonal Cutting Conditions" to "Analysis of Some Indentation Models Related to Incipient Deformation of Brittle & Ductile Workmaterials During Orthogonal Metal Cutting". The new title appears to more accurately accord with the contents of the thesis.
- (ii) Limitations of the proposed models vis-a-vis incipient deformation during actual metal cutting are discussed (Art. 6.5.5).
- (iii) The character of friction in metal cutting has been discussed in Article 1.2.4.
- (iv) Chapter - VI has been thoroughly revised. A unified analysis of the results of the proposed models has been presented. Units of force have been changed from kgf to N. Typical results predicted from the "extended quarter plane model", have been included.


(J.R. Anand)

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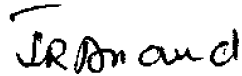
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ABSTRACT

Incipient deformation refers to the deformation suffered by the work-material just after the tool is brought into contact with the workpiece resulting in the formation of chip-head. Conditions giving rise to incipient chip formation occur most frequently in intermittent machining processes such as up- and down- milling, turning of splined or grooved shafts and machining of brittle workmaterials involving discontinuous chip formation. Even in the so-called steady or continuous chip formation, the nature of initial deformation is expected to exercise considerable effect on later deformation. Analysis of incipient deformation may prove valuable in obtaining quantitative results on the shape of the deformation zone in metal cutting, stress and strain distributions within the deforming zone as well as the cutting force. This knowledge may then be applied for securing a better understanding of tool wear, surface roughness and residual stresses in the machined surface and also machine tool vibration during intermittent cutting.

The problem of incipient deformation in metal cutting has attracted the attention of a number of research workers in the past. Ernest [57] made a photoelastic study of the transient stage by considering it analogous to the indentation of a semi-infinite medium by a sharp-edged punch. Field and Merchant [58] analysed it from the stand-point of discontinuous chip formation. Lee [59] analysed the problem of deformation prior to rupture using slip-line field theory. Weinmann et al and Weinmann [60, 61] made experimental study of incipient deformation again by wedge indentation. They interpreted the microhardness scans for studying the deformation pattern. The [62] investigated incipient cutting process using high speed photography.

The present work is devoted to the study of some indentation models related to incipient deformation during orthogonal metal cutting. Analytical solutions of the models are presented for elastic and non-linear strain hardening workmaterials for infinitesimal (linear) and small (elastoplastic) deformations respectively. For the case of large deformation, an experimental method based upon the Moire technique has been used.

For analysing the distributions of stress and strain during incipient chip formation, the workmaterial is idealized as a plate of uniform thickness acted upon by a rigid tool having a sharp cutting edge. The tool is given a side rake angle, and the length of cutting edge is much larger than the thickness of the plate. For the purpose of analysis, the work is considered to be a parallelopi-ed ed whose contacting face is parallel to the rake face of the tool. Three indentation models have been proposed for analysing deformation. Two of these are applica- ble to brittle workmaterials, one is based upon the classical theory of elasticity **and the other** on the Boundary Element Method. Solutions of stress and strain **distributions** in the workmaterial corresponding to different pressure and frictio- **nal stress** conditions at the rake face have been computed. In the solution based upon **the** boundary element method, stress and strain distributions corresponding to prescribed tool displacement have also been obtained. The third model is proposed **for** the case of elastic, non-linear strain hardening workmaterial. It is based on an updated Lagrangian finite element formulation for analysing elastoplastic deformation. Numerical results corresponding to different rake angles, frictional and loading conditions have been presented and discussed.

An experimental set-up for study of large deformation and resulting strain distribution was designed and fabricated. It utilized the Moire technique for measuring strain in the workmaterial during incipient chip formation. Experi- mental results are presented and discussed.

NOTATIONS

α	Tool rake angle
β	Tool-work friction angle
β^j	Angle between \bar{x} -axis of j^{th} element and x-axis
ϕ	Shear plane angle
ϕ_1^i, ϕ_2^i	Function relating to tangential and normal displacements and stresses respectively
ν	Poisson's ratio
ρ	Density
θ	Angles
δ	Displacements
ϵ, γ	Normal and shear strains
σ, τ	Stress components
ξ, η	Local coordinate system
λ	Lame's constant
Δ	Increment
G	Modulus of rigidity
E	Young's modulus
t	Workmaterial thickness
a^j	Half length of boundary element
x, y and \bar{x}, \bar{y}	Global coordinates system and local coordinates for boundary elements
u, v	Displacement in xy frame
P_s and P_n	Tangential and normal uniformly distributed line loads
$\bar{F}_1, \bar{F}_2, \bar{F}_3, \bar{F}_4, \bar{F}_5$	Influence coefficients
$A_{ss}^{ij}, A_{st}^{ij}, A_{ns}^{ij}$ & A_{nn}^{ij}	Influence coefficient for displacement
$B_{ss}^{ij}, B_{st}^{ij}, B_{ns}^{ij}$ & B_{nn}^{ij}	Influence coefficient for stresses
E^j	Boundary elements

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