ANALYTICAL SOLUTIONS FOR THREE DIMENSIONAL EDGE STRESS FIELD IN ELASTIC AND SMART PIEZOELECTRIC LAMINATES WITH WEAK INTERFACES

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by

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Submitted

in fullfillment of the requirements of the degree of Doctor of Philosophy

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Dedicated to
My Parents

Certificate

This is to certify that the thesis entitled "**Analytical Solutions for Three Dimensional Edge Stress field in Elastic and Smart Piezoelectric Laminates with Weak Interfaces**" being submitted by **Mr. Dhanesh N** to the Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy** in Applied Mechanics is a record of original bonafide research work carried out by him under my supervision and guidance. The thesis work, in my opinion, has reached the requisite standard fulfilling the requirements for the degree of Doctor of Philosophy.

The results contained in this thesis have not been submitted in part or full to any other University or Institute for the award of any degree or diploma.

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Abstract

Hybrid laminates made of composite or sandwich laminates with surface bonded or embedded piezoelectric layers are increasingly being used in light weight smart and multifunctional structures in advanced high performing applications such as aerospace vehicles with reconfigurable wings, variable twist marine propeller blades, wind turbine blades, flexible electronics, space antennas, and robotic arms. The laminates are characterized by discontinuity in mechanical, thermal and electric properties at the interfaces between dissimilar adjacent layers. This material discontinuity along with the geometric discontinuity at the edges of such laminated structures leads to the development of localized out-of-plane stresses at/near the interface near the edges, which decay rapidly away from the edges. It is commonly known as the edge effect, which needs to be accurately evaluated since it is known to be the major cause of initiation of delamination damage, which eventually may lead to premature structural failure and/or loss of actuation/sensing effectiveness. Another important problem faced by the laminates is the weakening of interfacial bonding during manufacturing and/or in service, which can affect the response.

In this thesis, an accurate three-dimensional (3D) elasticity solution is presented for static analysis of flat laminated panels with interlaminar bonding imperfection under arbitrary boundary conditions exhibiting edge effects. The recently developed mixed-field multiterm extended Kantorovich method (MMEKM) for 3D solution of perfectly bonded laminates is generalized to include the interfacial compliance characterized by displacement jumps. The bonding imperfection is modeled using the linear spring-layer model, which is incorporated into variationally consistent framework using the Reissner-type mixed variational principle. The solution has been further extended to piezoelectric laminates featuring weak interfacial bonding subjected to pressure and electric potential loading. The accuracy and convergence are demonstrated through comparison with the exact 3D solution for simply supported panels and a detailed FE analysis for other boundary conditions. The roles that the boundary conditions, locations of the imperfect interfaces, and span-to-thickness ratios play on the effect of weak bonding on the response of laminated structures are investigated for a wide range of values for the imperfection compliance. The effect of imperfection compliance on the dynamic response of elastic laminates, sensing and actuation capability of smart laminates are also investigated.

The free edge stress field is truly three dimensional and often singular in nature, whose accurate prediction has been the subject of intense research since the pioneering numerical work of Pipes and Pagano (1970). In this thesis, the MMEKM is employed to obtain accurate 3D piezothermoelasticity solution of the free edge stress field in infinitely long laminated panels under extension, bending, twisting, electric potential and thermal (uniform and gradient) loadings. The mechanical loadings are incorporated into the formulation in the form of applied strain and curvatures, using Lekhnitskii's displacement and strain fields (Lekhnitskii, 1963). The method ensures exact point-wise satisfaction of all boundary and interlaminar continuity conditions, which is key to obtaining accurate results for the localized stresses near free edges. The results are compared with various existing solutions reported for symmetric cross-ply and angle-ply laminates. New results are presented for the free edge stress field in antisymmetric angle-ply and soft-core sandwich laminates. The solution is further extended to include the effect of weak interfacial bonding. The presence of bonding imperfection at an interface leads to nonsingular free edge interlaminar stresses at that interface, in contrast with the singular stress field occurring at a classical perfect interface. The results show significant effect of electromechanical and pyroelectric coupling on the free edge stresses in hybrid laminates. The feasibility of reduction of free edge stresses caused by mechanical and thermal loads in hybrid laminates by the application of electric potential is also examined.

सार

दाबविध्युतिकीय परतों को सतह से बांधकर या अन्तः स्थापित करके बनाए गए सामासिक अथवा *सैंडविच* परतबंदी से बने *हाइब्रिड लेमिनेट्स* का उपयोग हल्के वज़न वाली *स्मार्ट* एवं बहुप्रयोजनमूलक संरचनाओं जैसे कि पंखों का प्रारूप बादल सकने वाले वांतिरक्ष वाहन, परिवर्तनशील घुमावदार पत्तियों वाले समुद्री *प्रोपेलर,* पवन चक्की *टर्बाइन* की पत्तियाँ, लचीले *इलेक्ट्रोनिक,* अन्तरिक्ष *एंटीने* और यांत्रिक अंग में तेज़ी से बढ़ रहा है। लेमिनेट्स का मूल्यांकन असमान पाशवस्थ परतों के अन्तराफलक की यांत्रिक समाप्ती, तापीय और विधुतीय विशेषताओं के आधार पर होता है। किनारों पर पदार्थ की ज्यामितीय समाप्ती की वजह से, एसी परतबंद संरचनाओं में, किनारों के पास या किनारों के अन्तरफलक पर स्थानबद्ध सतह से बाहर तनाव उत्पन्न होता है, जिसकी वजह से किनारों से दूर क्षय बढ़ता चला जाता है। इसे समान्यतः किनारे के प्रभाव की तरह जाना जाता है, इसका सटीक मूल्यांकन करना ज़रूरी है क्योंकि यह गैर-परतबंदी क्षति की शुरुआत का मुख्य कारक है, जो अंततः प्रवर्तन/ संवेदन प्रभावशीलता की हानि और/ अथवा समयपूर्व संरचनात्मक विफलता का कारण बन सकता है। निर्माण और/ अथवा सेवा के दौरान अन्तराफलक बंधनो का कमजोर हो जाना, एक और अहम समस्या है जिसका सामना लेमिनट्स को करना पड़ता है और जो प्रतिक्रिया को प्रभावित कर सकती है।

इस शोध प्रबंध में, अंतरफलक बंधनो की अपूर्णता के साथ वाले एसे सपाट पट्टे जो मनमानी परिस्थितियों में किनारा प्रभाव प्रदर्शित करते हैं, के स्थिर विष्लेषण हेतु एक सटीक त्रि-आयामी लचीला समाधान प्रस्तुत किया गया है। संपूर्णतः बंधे हुए *लेमिनट्स* के त्रि-आयामी समाधान हेतु, हाल ही में विकसित मिश्रित-क्षेत्र बहु-अवधि विस्तृत कांटोरोविच विधि (एम॰एम॰ई॰के॰एम) का, विस्थापन कूद की विशेषता रखने वाले अन्तराफलक अनुपालन को शामिल करने हेतु, सामान्यीकरण किया गया है। बंधनो की अपूर्णता को रेखाकार *स्प्रिंग-लेयर* आदर्श के उपयोग से गढ़ा गया है, जिसे रेइस्स्नेर-प्रकार मिश्रित परिवर्तनशील सिद्धांत का उपयोग करते हुए, परिवर्तनशील सुसंगत ढांचे में निगमित किया गया है। दबाव और सशक्त विधुतीय लदान की स्थिति में कमज़ोर अंतरफलक बंधनो का प्रदर्शन करने वाले दाबविध्युतिकीय लेमिनट्स को शामिल करने हेतु इस समाधान का आगे विस्तार किया गया है। सटीकता और अभिसरण का प्रदर्शन, सरल समर्थित पट्टों के सटीक त्रि-आयामी समाधान के साथ की गयी तुलना और दूसरी परिसीमा प्रतिबंध के विस्तृत एफ॰ई विश्लेषण, के माध्यम से किया गया है। अपूर्णता अनुपालन हेतु, परतबंद संरचनाओं की प्रतिक्रिया पर, कमज़ोर बंधनों के प्रभाव में - परिसीमा प्रतिबंध, अपूर्ण अंतराफलकों के स्थान और विस्तार से मोटाई-के-अनुपात जो भूमिका निभाते हैं, उसकी एक वृहद मानों वाली श्रृंखला के लिए जांच की गयी है। लचीले *लेमिनट्स* की

गतिशील प्रतिक्रिया पर अपूर्णता अनुपालन के प्रभाव, *स्मार्ट लेमिनट्स* की संवेदन और प्रवर्तन क्षमता की भी जाँच की गयी है।

मुक्त किनारों का तनाव क्षेत्र वास्तव में त्रि-आयामी है और अक्सर प्रथक प्रकृति का है, इसकी सटीक पूर्व-सूचना, पाइप्स और पैगानो (१९७०) के अग्रणी संख्यात्मक कार्य के समय से ही, एक गहन शोध का विषय रही है। इस शोध प्रबंध में, एम॰एम॰ई॰के॰एम का प्रयोग - विस्तार, झुकाव, घुमाव, विध्युतीय क्षमता और तापीय (समरूप और उतार-चढ़ाव वाली) लदान के आधीन अनंत लंबे परतबंद पट्टों में मुक्त किनारों के तनाव क्षेत्र के सटीक त्रि-आयामी दाब-तापीय-लचीले समाधान हेतु किया गया है। यांत्रिक लदान को प्रयुक्त खिंचाव और वक्रता के रूप में, लेखनिट्सकी के विस्थापन और खिंचाव क्षेत्र (लेखनिट्सकी, १९६३) का उपयोग करके, सूत्रीकरण में शामिल किया गया है। यह प्रक्रिया सभी परिसीमा और अंतःपटलीय निरंतर प्रतिबंधों की सटीक बिंदुवार संतुष्टि सुनिश्चित करती है, जो कि मुक्त किनारों के निकट स्थानबद्ध तनाव के सटीक परिणामों को प्राप्त करने की कुंजी है। नतीजों की तुलना सममित *क्रॉस-प्लाई* और *एंगल-प्लाई लेमिनट्स* के लिए उपलब्ध विभिन्न मौजूदा समाधानों के साथ की गयी है। असममितीय *एंगल-प्लाई* और नरम-मूल वाले *सैंडविच लेमिनेट्स* में, मुक्त किनारा तनाव क्षेत्र के लिए नए परिणाम प्रस्तुत किए गए हैं। कमज़ोर अन्तराफलक बंधनों के प्रभाव को शामिल करने हेतु इस समाधान का और विस्तार किया गया है। बंधनों कि अपूर्णता की उपस्थिति अंतरफलक पर, मान्य दोषहीन अंतरफलक में उत्पन्न होने वाले प्रथक तनाव से उलट, व्युत्क्रमणीय मुक्त किनारों वाला अंतःपटलीय तनाव उत्पन्न कर देती है। यह परिणाम विद्युत-यांत्रिक और अग्निमैध्युत युग्मकों का *हाइब्रिड लैमिनेट्स* में, मुक्त किनारे वाले तनावों पर महत्वपूर्ण प्रभाव दर्शाता है। *हाइब्रिड लेमिनेट्स* में, विधुतीय क्षमता के प्रयोग से यांत्रिक और तापीय दबाव से उत्पन्न मुक्त किनारे वाले तनावों में कमी की संभाव्यता भी जाँची गयी है।

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