Subdifferential Characterization of Approximate Solutions in Optimization Problems

By

Deepali Gupta

Department of Mathematics

Submitted

in fulfillment of the requirements

of the degree of Doctor of Philosophy

to the

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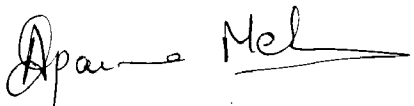
To My Parents

&

Late Prof. M.C. Puri

Certificate

I am satisfied that the thesis entitled **Subdifferential Characterization of Approximate Solutions in Optimization Problems** presented by **Ms. Deepali Gupta** (2004MAZ8059) is worthy of consideration for the award of the degree of Doctor of philosophy and is a record of the original bonafide research work carried out by her under my guidance and supervision and the results contained in it have not been submitted in part or full to any other university or Institute for award of any degree/diploma.



Dr. Aparna Mehra

Supervisor

Department of Mathematics

Indian Institute of Technology Delhi

Hauz Khas, New Delhi-110016

India

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Deepali Gupta

New Delhi Deepali Gupta

Abstract

Vector optimization concern with making decisions to balance various conflicting objectives via optimally distributing scarce resources. However, many realistic problems are not blessed with a single decision/solution optimizing all objectives. This has led to various solution concepts, each having distinct features, for vector optimization problems. Among many existing concepts, the one which is most widely used is the concept of efficient solutions.

The set of exact efficient solutions is costly to generate and often unattainable, or there are necessary approximations in modeling of the real world problems. In such scenario, the set of approximate efficient solutions becomes an appealing alternate. The concept of approximate efficiency can be considered as a satisfactory compromise with a prescribed error in the efficient values of the vector objective function. Approximate efficient solutions are found to be theoretically as well as computationally useful. The aim of the present research is to study approximate efficient solutions for two important classes of optimization problems, namely, the vector optimization problems, and the semi-infinite programming problems. The initial focus in the present work is concerning the vector optimization problems (VOP) while later we shift our attention to study the semi-infinite programming problems (SIP).

There are broadly two approaches to deal with VOP. Firstly, to study the problem as

a vector problem and secondly to study the problem via scalarization. We adopted both approaches in our work to study various types of approximate solutions for constrained VOP and characterize them via the Lagrangian saddle point or the Karush Kuhn Tucker (KKT) optimality conditions. Further, for scalar constrained SIP, we define a new concept of approximate optimal solutions. Characterizations of this solution concept in terms of the KKT optimality conditions are worked out via two different sequential approaches, namely, the penalty function approach and by introducing perturbations in the feasible sets of the original problem.

Throughout the thesis, the Ekeland's Variational Principle and penalty function approach have been effectively used to study characterization of various approximate solutions for different optimization problems. Furthermore, during entire course of our discussion, we have taken aid of some concepts from nonsmooth optimization, convex analysis and variational analysis.

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