APPROXIMATION OF P-STRUCTURE, NONLOCAL KIRCHHOFF AND MHD PROBLEMS USING WEIGHTED EXTENDED B-SPLINE BASED FEM

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by

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Dedicated to

My Family

Certificate

This is to certify that the thesis entitled **"Approximation of p-structure, nonlocal Kirchhoff and MHD problems using Weighted Extended B-Spline based FEM"** submitted by **"Mr. Sudhakar"** to the Indian Institute of Technology Delhi, for the award of the Degree of Doctor of Philosophy, is a record of the original bona fide research work carried out by him under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted in part or full to any other university or institute for the award of any degree or diploma.

New Delhi
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Abstract

Partial differential equations (PDEs) have many important applications in various fields of science and engineering, such as fluid dynamics, materials science, inverse problems, computer graphics, and pattern formation. However, analytic solution of these type of problems is either not known or very difficult to develop. Therefore, one should look for accurate and efficient numerical methods to solve such type of problems. Finite element method is one of the numerical techniques which may be used to solve the time dependent problems as well as steady state problems. The classical finite element method uses low order piecewise polynomials defined on polygonal domains. This thesis comprises both the theoretical and numerical results obtained for linear and non-linear problems using web-spline based FEM. The domains are discretized into simple polygons called the mesh. These polygons might be triangles, quadrilaterals, etc., for two-dimensional domains, and tetrahedra, hexahedra, etc., for three-dimensional domains. In classical two-dimensional finite element methods, the basis functions are usually hat functions defined on triangulations. Another possible selection of a finite element basis in two dimensions are tensor product b-splines. In engineering applications, the description of geometry and mesh generation are often the most time consuming process in finite element methods. It is presently a marked interest for meshless methods as a tool for solving complex boundary-value problems in science and engineering. Choe et al. [19, 20] has investigated the use of moving least squares reproducing kernel in the Galerkin formulation of Navier-Stokes equations. Weighted extended B-spline based mesh-free methods for boundary value problems are proposed as a natural generalization of standard B-splines to address two important issues of exact fulfilment of the boundary conditions and well-conditioning of the Galerkin system. The development of this new strategy for the Laplace operator problem is due to Hollig et al. [40–42]. The concepts of **weight** and **extension procedure** will be described in the next chapter.

Motivated by the fact that Laplace operator (in Poisson problem) is a special case of p-Laplace operator (in p-Laplacian problem) and web-spline method is well developed for linear elliptic PDEs, in this thesis, we have proposed web-spline method for quasi-linear p-Laplace problem. We also have made some numerical experiments to show the convergence of the numerical solution.

Further we have solved non-local problem of Kirchhoff type using web-spline based mesh free finite element method. We also have derived web-spline based a priori error estimates.

Authors V.V.K Srinivas et al. have studied stationary Stokes and Navier-Stokes problems [49]. With this motivation we proposed web-spline method for time dependent heat equation and Navier-Stokes equation. Error estimates are also being provided.

We extend the web-spline based mesh free finite element results from linear Stokes problem [17, 49–51] to p-Stokes problem (quasi-Newtonian flow problem). We prove the well-posedness of mixed formulation for p-Stokes at continuous level as well as at the discrete mixed formulation. We discuss the solvability of the matrix system arising after applying web-spline method and have given a priori error estimates for the solution of quasi-Newtonian flow problems based on web-spline method.

We also give a new class of web-spline based finite elements for the stable approximation of Maxwell equations.

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