PROGRAMMING TECHNIQUES FOR REMOVING DATA DEPENDENCIES IN GENERALISED EIGEN PROBLEMS WITH APPLICATIONS

By

JOSEPH KURIAN

A Thesis
submitted for fulfilment of
the requirement of

DOCTOR OF PHILOSOPHY

To the

INDIAN INSTITUTE OF TECHNOLOGY, DELHI

NEW DELHI 110 016

February 1983

CERTIFICATE

Certified that the thesis entitled "PROGRAMMING TECHNIQUES FOR REMOVING DATA DEPENDENCIES IN GENERALISED EIGEN-PROBLEMS WITH APPLICATIONS" being submitted by Mr. Joseph Kurian, is a record of work carried out by him under my supervision. The matter embodied in this thesis has not been submitted for the award of any other degree.

(Dr. P.G. Reddy)
Professor
Computer Service Centre
Indian Institute of Technology Delhi
New Delhi-16

ACKNOWLEDGEMENT

I am grateful to my thesis supervisor, Dr. P.G. Reddy, Professor, Computer Centre, Indian Institute of Technology, Delhi, for his valuable guidance, unstinting help and stimulating encouragement. I am thankful to Dr. M.K. Jain, Professor and Head, Computer Centre, for the facilities provided in the Centre. I am indebted to the authorities of the Indian Institute of Technology, Delhi, for the opportunity provided at the Institute to carry out this research work.

I express my sincere gratitude to Dr. K. Athre, Lecturer, Department of Mechanical Engineering, for his keen interest in the investigation and its applications. He has been a great source of inspiration during the course of this work.

I am indebted to Dr.Ing.- K.N. Gupta, Professor, Department of Mechanical Engineering for suggesting the problem of Chapter 5 and providing the numerical examples. I acknowledge the help of Dr. S.R.K. Iyengar, Professor, Department of Mathematics, through many hours of discussions during the course of this work.

The devotion, encouragement and patience of my wife Mrs. Susy Joseph, and the forberance of my daughter Anshu Annie Joseph, made this work a possibility.

I am thankful to Mr. P.M. Padmanabhan Nambiar and Miss Neelam Dhody for their neat typing of this thesis.

Finally, I would like to dedicate this work to the memory of my father Chettukandathil Joseph Kurian, who consistently inspired me to seek higher education at every available opportunity.

JOSEPH KURIAN

ABSTRACT

Matrix equations of the form,

$$\left[A^{(1)}\lambda^{m} + A^{(2)}\lambda^{m-1} + \cdots + A^{(i)}\lambda^{m-i+1} + \cdots + A^{(m)}\lambda + A^{(m+1)} \right] X = 0$$

where $A^{(i)}$'s are n x n matrices, λ the eigenvalue, X the eigenvector and m a positive integer, are quite common in the study of Linear Vibrations of Dynamic Systems, Information System Design, Control System Theory, Nonlinear Optimization, Mathematical Economics, and numerous other Engineering Applications and Mathematical Problems [1,2].

$$[A - \lambda I]\, X = 0 \quad \text{and}$$

$$[A - \lambda B]\, X = 0 \ ,$$

are special cases of the above generalized eigen problem [1,2], where A and B are n x n matrices, and I the identity matrix. The generalized eigen problem has non-trivial solutions if the characteristic determinant is zero, i.e.,

$$\left| A^{(1)}\lambda^{m} + A^{(2)}\lambda^{m-1} + \cdots + A^{(i)}\lambda^{m-i+1} + \cdots + A^{(m)}\lambda + A^{(m+1)} \right| = 0 \qquad (1)$$

In the quadratic case (when m = 2) using A, B and C for representation of the matrices and corresponding small case letters for the representation of matrix elements, the characteristic determinant can be written as

$$\left| A\lambda^{2} + B\lambda + C \right| = 0 \qquad (2)$$

or more explicitly as

$$\begin{vmatrix} a_{11}\lambda^2 + b_{11}\lambda + c_{11} & a_{12}\lambda^2 + b_{12}\lambda + c_{12} & a_{13}\lambda^2 + b_{13}\lambda + c_{13} \\ a_{21}\lambda^2 + b_{21}\lambda + c_{21} & a_{22}\lambda^2 + b_{22}\lambda + c_{22} & a_{23}\lambda^2 + b_{23}\lambda + c_{23} \\ a_{31}\lambda^2 + b_{31}\lambda + c_{31} & a_{32}\lambda^2 + b_{32}\lambda + c_{32} & a_{33}\lambda^2 + b_{33}\lambda + c_{33} \end{vmatrix} = 0 \tag{3}$$

In many practical problems, especially in the study of stability characteristics of dynamic mechanical systems, it is sufficient to know the coefficients p_j of the characteristic polynomial,

$$\sum_{j=0}^{n\times m} p_j \lambda^j = 0 \tag{4}$$

obtained from the characteristic determinant (1). The construction of (4) from (1) is a problem of great interest. The majority of methods which construct the characteristic polynomial of the generalized eigen problem are based on approximations or iterative in nature. According to Someya [3],

> "Because of the high degree of freedom the algebraic determination of the coefficients of the characteristic equation was very difficult and a numerical method was used. The value of the determinant was calculated for different values of λ , for the number of coefficients of the characteristic equation, and linear equations for C_i were solved. To the characteristic equation so derived Routh stability criteria were applied. The accuracy of the results depends in some cases heavily on the accuracy of C_i."

C_i's are the coefficients of the characteristic equation (polynomial). Choudhury and Gunter [4] have given two approximate methods, viz. the method of interpolation and the method of undetermined coefficients.

Exact values are not obtained by these methods as truncation error is involved. A computer program by Hennion [5] is available in ALGOL, wherein the matrix is reduced to the upper triangular form and polynomial multiplication of the elements on the diagonal done. This procedure may produce a lot of numerical inaccuracy as simultaneous triangularization of more than one matrix is involved. According to Vernon [6] who succeeded in row (column) expansion of the quadratic characteristic determinant equation (3).

> "The efficient expansion by digital computer of a high order determinant of quadratics of the form shown in 1-47 is an EXERCISE IN LOGIC."

(Equation 1-47 refers to the one given in [6]),

Construction of (4) by expanding (1) and evaluating coefficient determinants of powers of λ is a direct method. This method is seldom attempted because of the computational complexities. The essence of this doctoral thesis is the development of various algorithms for the expansion of (1) and construction of (4) overcoming all computational complexities/difficulties and ensuring numerical accuracy. Every effort is made to make the algorithm most general, simple, and easy to understand. Since the bulk of the users of such algorithms are Scientists and Engineers, complete software is developed in FORTRAN IV for easy usage.

Expansion of (1) is achieved through a combinatorial enumeration [7] process of the coefficient determinants.Normally, matrices are stored in FORTRAN IV in two dimensional arrays, and enumeration is achieved by the use of nested loops. Such an algorithm, if attempted, is data dependent, i.e. the algorithm is dependent on the order n of the matrices and the degree m of the polynomial elements. In other words, one has to write different programs for different values of n and different values of m, as there will be n nested loops and m+1 different matrices. Such a program is not general. Therefore, it is of importance to have a proper data structure [8] to store the coefficient matrices of (1). The simplest data structure available for data storage in FORTRAN IV is linear arrays. In the developed algorithms, coefficient matrices of (1) are stacked in linear arrays, column by column, and the enumeration process is achieved by reducing the n nested loops to a single loop. For large values of n and m, the enumeration process results in a 'combinatorial explosion'. Therefore, efficient handling of this explosion is vital. In practical problems, it is observed that the coefficient matrices encountered are sparse. The combinatorial explosion is, therefore, efficiently handled by exploiting sparsity. The algorithms developed are : (i) data dependent, (ii) data independent, and (iii) data independent sparse.

The algorithms are applied to solve practical engineering problems. In the study of stability of single floating bush bearings, the equation of equilibrium leads to a 6 x 6 characteristic determinant equation [9,10] with polynomial elements in quadratics. In this case, results evaluated through analytical expansion of the determinant equation is available for comparison. The results obtained by the developed algorithms are compared with the analytically obtained results and excellent agreement is noticed. An error in one of the coefficients obtained by Tanaka and Hori [10] during analytical expansion is located. A modified design of the aforesaid bearing is the double floating bush bearing [9]. In this case the equations of equilibrium lead to an 8 x 8 characteristic determinant equation with quadratic elements, where analytical evaluation is extremely difficult. Because of the data independence of the developed algorithm, same computer program is used to solve this problem also.*

There is a class of problems in structural dynamics which are solved by Transfer Matrix Method, where the characteristic determinant itself is to be constructed as a product of transfer matrices with polynomial elements. Methods developed by Myklestad and Prohl [12,13] are extensively used for computing the eigen frequencies of a complex beam structure. By this method the state vector is progressively computed from one station to the next and the eigen value λ is determined by successive iteration such that it satisfies the state vector at the boundaries. Rao and Banerjee [14] tried to develop the characteristic polynomial from these transfer matrices and succeeded in the formulation of the coefficients of the polynomial only in the cases of some problems of cantilever blade vibrations. This formulation lacks in their universal application to any transfer matrix problems. Since each individual transfer matrix

---

* Part of the above work is published in Trans. ASME, Journal of Mechanical Design [11].

has its elements as polynomials, a technique is needed, which may generate a product of such matrices, and finally determine the characteristic polynomial. This would certainly reduce the iteration time, otherwise involved. In this thesis, a data independent algorithm is also developed for the construction of such a characteristic determinant. Some numerical examples for which results are available in literature are solved by this new algorithm and the results are found to be in good agreement. In case, the eigenvalues are required the same can be obtained by finding the zeros of the characteristic polynomial using the Jenkins algorithm [15]. All the algorithms developed in the thesis can be converted easily to handle problems in complex variables.

CONTENTS

CONTENTS (CONTD.)

CONTENTS (CONTD.)