

IMPACT RESPONSE OF VISCOELASTICALLY
DAMPED LAMINATED STRUCTURES

by

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ABSTRACT

The partial differential equations describing the transient vibrations of the constrained and unconstrained configurations of the laminated beams, having elastic and viscoelastic layers, have been derived. Impact loading in the form of half-sine pulse and rectangular pulse has been taken and solutions have been obtained to find the impact response of the laminated beams, by assuming infinite sinusoidal series solution for space variable and Laplace transformation technique or exponential solution for time variable. Three element and four element models are used to represent the properties of the viscoelastic layer. The solutions have been given for simply supported laminated beams. The theory given for three layered case is applicable to any unsymmetrical configuration. Detailed analysis for the three layered and two layered arrangements includes obtaining the solutions for transient response, studying the influence of excitation period, non-dimensional geometrical and shear parameters, unsymmetry, ratio of the thickness of viscoelastic layer to elastic layer and the effect of the damping constants of the viscoelastic model on the impact effectiveness of the structure. The effectiveness has been designated by the maximum response and by the decay of response after the pulse has ceased to act. The

equation of each system obtained by theoretical analysis have been programmed on ICL 1909 digital computer. For the viscoelastic material, model constants have been determined from properties which were obtained under harmonic excitation conditions. The performance of the three layered beam with viscoelastic core has been compared with that of an undamped sandwich beam with corresponding elastic core. Further comparisons have been made between the impact performance of three layered sandwich beam with elastic core and a homogeneous beam and between three and two layered configurations. Impact tests have been carried out on a few samples of laminated beams, in order to verify the theoretical results of three layered and two layered configurations.

CERTIFICATE

This is to certify that the thesis entitled 'Impact Response of Laminated Structures' being submitted by Mr. Des Raj Chawla to the Indian Institute of Technology, Delhi, for the award of the Degree of Doctor of Philosophy in Mechanical Engineering, is a record of bonafide research work carried out by him. He has worked under my guidance and supervision and has fulfilled the requirements for the submission of this thesis, which has reached the requisite standard.

The results contained in this thesis have not been submitted in part or in full, to any other university or institute for the award of any degree or diploma.

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