NONLINEAR DYNAMICS OF BIMODULAR LAMINATED COMPOSITE PANELS

by

ARSHAD HUSSAIN KHAN

Department of Applied Mechanics

Submitted
in fulfillment of the requirements of the degree of

DOCTOR OF PHILOSOPHY

to the

INDIAN INSTITUTE OF TECHNOLOGY DELHI
APRIL 2015

© Indian Institute of Technology Delhi (IITD), New Delhi, 2015

NONLINEAR DYNAMICS OF BIMODULAR LAMINATED COMPOSITE PANELS

by

ARSHAD HUSSAIN KHAN

Department of Applied Mechanics

Submitted
in fulfillment of the requirements of the degree of

DOCTOR OF PHILOSOPHY

to the

INDIAN INSTITUTE OF TECHNOLOGY DELHI
APRIL 2015

Certificate

This is to certify that the thesis entitled **“Nonlinear Dynamics of Bimodular Laminated Composite Panels”** being submitted by **Mr. Arshad Hussain Khan** to the Indian Institute of Technology Delhi for the award of the degree of **Doctor of Philosophy** in Applied Mechanics is a record of original, bonafide research work carried out by him under my supervision and guidance. The thesis work, in my opinion, has reached the requisite standard fulfilling the requirements for the degree of Doctor of Philosophy.

The results contained in this thesis have not been submitted in part or in full, to any other University or Institute for the award of any degree or diploma.

Date:
New Delhi

(Dr. B. P. Patel)
Professor
Department of Applied Mechanics
Indian Institute of Technology Delhi
New Delhi- 110016
India

Acknowledgements

I would like to extend my deep sense of gratitude and appreciation to my supervisor Prof. B. P. Patel for his expert guidance, inspiration, suggestion and continuous encouragement. His invaluable inputs, stimulating discussions and intellectual support at every stage of my research contributed to the completion of this work. My association with him has been a great learning experience.

My appreciation and thanks are also expressed to my research committee members for their review, discussions and suggestions. I would also like to express my thanks and gratitude to the Head and faculty members of Department of Applied Mechanics, Indian Institute of Technology Delhi for their kind help and support.

I am immensely thankful to my friends and fellow research scholars of the department for their wonderful company, generous help and for making my stay in Indian Institute of Technology Delhi a very enjoyable and enriching experience. I also want to thank all who have directly or indirectly helped me during the course of this work.

I would also like to gratefully thank my colleagues and senior faculty members of Mechanical Engineering Department, AMU, Aligarh for their encouragement, support and help.

I have no words to express my thanks to my father and late mother for shaping my career and for their endless love and affection which has been a constant source of inspiration

for me. Thanks are also due to my brothers and sisters for their constant encouragement and moral support. I am also thankful to my in-laws for their appreciative support and help.

Last but not the least, I am highly indebted to my wife and kids Aniqa, Aliza and Azlan for their endless love, moral support and inspiration for the accomplishment of this work.

(Arshad Hussain Khan)

Abstract

Many of the composites such as aramid-rubber, polyster-rubber, carbon-carbon, soft biological tissues, bone referred to as bimodular composites exhibit different elastic properties in tension and compression. The dynamic analysis of bimodular structures is computationally challenging due to the non-smooth nonlinear nature of governing equations with elastic moduli dependent on the sign of stress/strain which is unknown *a priori*. For thin structural components undergoing transverse displacements of the order of their thickness, the consideration of geometric nonlinearity becomes important for accurate prediction of deflections, strains, stresses and response frequencies. It is concluded from the literature review that the periodic response characteristics of bimodular laminated plates/panels including geometric nonlinearity have not been investigated. An attempt is made in the thesis to address the problem using the first-order shear deformation theory based finite element model incorporating bimodularity through Bert's constitutive model and geometric nonlinearity. The governing equations of motion are solved to obtain the state vector representing the periodic response by employing shooting technique coupled with Newmark time marching and arc length/pseudo-arc length continuation algorithms.

The combined influence of bimodularity and geometric nonlinearity on the steady state periodic response characteristics of bimodular material laminated composite rectangular/annular sector plates and cylindrical/conical panels is analysed for the first time and several new results are presented. The stable and unstable portions of the frequency response and the associated bifurcations have been identified. The temporal and through the thickness stress/strain variations, frequency spectra of the steady state displacement/stress histories, phase plane plots and strain energy contributions from linear, quadratic and cubic restoring forces are presented to explore the non-linear dynamic characteristics. The results presented herein, without *apriori* assumptions of participating modes and harmonics, may also serve as reference for validation of approximate analytical solutions. Significant differences in the positive/negative half cycle response/stress amplitudes with unequal time spent in the positive/negative half cycle are predicted for bimodular plates/panels. Further significant differences in the response/stress amplitudes are predicted with and without geometric nonlinearity. Steady state frequency response of bimodular laminated rectangular/annular sector plates reveal hardening nonlinear behaviour for all the cases whereas the cylindrical/conical panels exhibit hardening or softening behaviour depending upon the parameters and boundary conditions. It is predicted that the hardening behaviour changes to softening one for cylindrical/conical panels with the decrease in the radius to thickness ratio. The degree of hardening nonlinearity increase with the increase in bimodularity ratio, aspect ratio, thickness ratio, sector angle, semi-cone angle and load amplitude. The GNL frequency response in some cases reveals strong modal interactions between first and third modes, and the presence of secondary branch emanating from the primary one through period doubling bifurcation. The radially outward imperfection results in increased hardening and the inward one in reduced hardening behaviour for cylindrical panels.

Contents

Certificate *i*

Acknowledgements *ii*

Abstract *iv*

Contents *vi*

List of Figures *ix*

List of Tables *xv*

Nomenclature and Abbreviations *xvii*

1. Introduction and Literature Review **1**

- 1.1 Introduction 1
- 1.2 Literature Review 4
  - 1.2.1 Constitutive Models 4
  - 1.2.2 Static Analysis of Bimodular Plates and Shells 5
  - 1.2.3 Dynamic Analysis of Bimodular Plates and Shells 9
    - 1.2.3.1 Free Vibration Analysis 9
    - 1.2.3.2 Transient Response Analysis 11
    - 1.2.3.3 Steady State Response Analysis 12
  - 1.2.4 Dynamic Analysis of Unimodular Annular Sector Plates 12
- 1.3 Motivation and Objectives 13
- 1.4 Organization of the Thesis 17

2. Formulation and Solution Methodology **19**

- 2.1 Finite Element Formulation 19
- 2.2 Solution Methodology 27
- 2.3 Validation 31
  - 2.3.1 Nonlinear Periodic Response of Unimodular Isotropic Plate 33
  - 2.3.2 Nonlinear Periodic Response of Unimodular Laminated Composite Cylindrical Panel 34
  - 2.3.3 Free Vibration of Isotropic/Orthotropic Unimodular Annular 35

Sector Plates
2.3.4 Free Vibration Frequencies of Bimodular Laminated Cylindrical Panels 37
2.4 Concluding Remarks 37
3. Nonlinear Periodic Response of Bimodular Laminated Composite Plates 38
3.1 Movable/Immovable Simply Supported Plates 42
3.2 Immovable Clamped ($C_IC_IC_IC_I$) and Immovable Clamped-Movable Simply Supported ($C_IS_MC_IS_M$) Plates 52
3.3 Concluding Remarks 68
4. Nonlinear Periodic Response of Bimodular Laminated Composite Annular Sector Plates 70
4.1 Effect of Bimodularity, Boundary Condition and Load Amplitude 73
4.2 Effect of Number of Layers 80
4.3 Influence of Fiber Angle/Lamination Scheme 81
4.4 Influence of Sector Angle 83
4.5 Concluding Remarks 92
5. Nonlinear Periodic Response of Bimodular Laminated Composite Cylindrical Panels 94
5.1 Influence of Bimodularity, Load Amplitude and Boundary Condition 97
5.2 Influence of Aspect Ratio, Radius to Thickness Ratio and Lamination Scheme 107
5.3 Effect of Initial Imperfection 113
5.4 Concluding Remarks 114
6. Nonlinear Periodic Response of Bimodular Laminated Composite Conical Panels 116
6.1 Effects of Load Amplitude and Boundary Conditions 119
6.2 Effect of Semi-Cone Angle 127
6.3 Effects of Aspect Ratio, Thickness Ratio and Number of Layers 133
6.4 Concluding Remarks 137

7. Conclusions and Scope for Future Research Work **139**

7.1 Conclusions from Present Work 140

7.2 Scope for Future Research 143

References **144**

List of Publications from the Thesis **156**

Bio-Data of the Author **157**

List of Figures

Fig. 1.1 Typical stress strain curve of bimodular materials. **1**

Fig. 2.1 Geometry and coordinate system of laminated conical panel. **20**

Fig. 2.2 Comparison of frequency response curves of Aluminium plate for different excitation amplitudes and damping. **34**

Fig. 2.3 Comparison of steady state response history at the center of the eight layered unimodular laminated cylindrical panel (*L*/*h* = 50, *b*/*r* = 0.4). **35**

Fig. 3.1 Geometry and coordinate system of bimodular composite laminated rectangular plate. **39**

Fig. 3.2 Frequency response curves of bimodular cross-ply laminated $S_MS_MS_MS_M$ plates ($\varsigma_1 = 0.01$). **43**

Fig. 3.3 Phase plane plots, frequency spectra and through the thickness fiber direction normal strain distribution of two-layered ($0^0/90^0$) bimodular laminated $S_MS_MS_MS_M$ plate (*L/b* =2, *b*/*h* = 20, q_0= 200 Pa, $\varsigma_1 = 0.01$). **47**

Fig. 3.4 Phase plane plots and frequency spectra of bimodular laminated $S_MS_MS_MS_M$ plate (*L/b* =0.5, *b*/*h*=50, q_0= 100 Pa, $\varsigma_1 = 0.01$) with geometric nonlinearity. **48**

Fig. 3.5 Frequency response curves and GNL steady state response history, phase plane plots and frequency spectra of two-layered ($0^0/90^0$) bimodular laminated movable ($S_MS_MS_MS_M$) and immovable ($S_IS_IS_IS_I$) simply supported plates (*L/b* = 1, *b/h* =10, q_0= 5 kPa, $\varsigma_1 = 0.01$, Material 1). **50**

Fig. 3.6 Frequency response curves of two-layered cross-/angle-ply bimodular laminated movable ($S_MS_MS_MS_M$) simply supported **51**

plates ($L/b = 1$, $b/h = 10$, $q_0 = 5$ kPa, $\varsigma_1 = 0.01$, Material 1).

Fig. 3.7 Frequency response curves of bimodular laminated $C_IC_IC_IC_I$ and $C_IS_MC_IS_M$ plates ($0^0/90^0$, $b/h = 100$, $q_0 = 10$ Pa, $\varsigma_1 = 0.05$, Material 1). **53**

Fig. 3.8 Displacement variations corresponding to the points marked in the frequency response curve of Fig. 3.7 (b) ($C_IS_MC_IS_M$, $L/b = 2$). **54**

Fig. 3.9 Frequency response curves of bimodular cross-ply $(0^0/90^0)_N$ laminated $C_IC_IC_IC_I$ plates ($L/b = 1.0$, $b/h = 100$, $q_0 = 10$ Pa, $\varsigma_1 = 0.01$, Material 1). **56**

Fig. 3.10 Frequency response curves of bimodular laminated cross-ply $(0^0/90^0)_N$ $C_IC_IC_IC_I$ plates ($L/b = 1.0$, $b/h = 100$, $q_0 = 10$ Pa, $\varsigma_1 = 0.01$, Material 2). **57**

Fig. 3.11 Phase plane plots and frequency spectra of the steady state response corresponding to GNL frequency response curves of Fig. 3.10. **58**

Fig. 3.12 Frequency response curves of bimodular laminated eight layered $C_IC_IC_IC_I$ plate ($(0^0/90^0)_4$, $b/h = 100$, $q_0 = 100$ Pa, $\varsigma_1 = 0.01$, Material 1). **60**

Fig. 3.13 Steady state response history, phase plane plots and frequency spectra corresponding to GNL frequency response curves of Fig. 3.12 ($L/b = 2$). **61**

Fig. 3.14 Displacement variations corresponding to the frequency response curve of Fig. 3.12 ($L/b = 2$). **62**

Fig. 3.15 Through the thickness fiber direction normal strain (ε_{11}) distribution corresponding to frequency response curve of Fig. 3.12. **63**

Fig. 3.16 Frequency response curves of bimodular laminated cross-ply **65**

$C_IC_IC_IC_I$ plate ($0^0/90^0$, L/b = 2, b/h =100, q_0= 100 Pa, $\varsigma_1 = 0.01$, Material 3).

Fig. 3.17 Steady state response history, phase plane plots, frequency spectra and displacement variations corresponding to GNL frequency response curves of Fig. 3.16. **66**

Fig. 3.18 Steady state response history and through the thickness fiber direction normal strain distribution corresponding to peak points in the GNL frequency response curves (Fig. 3.16, E_{2t}/E_{2c}= 0.5). **67**

Fig. 4.1 Geometry and coordinate system of laminated composite annular sector plate. **71**

Fig. 4.2 Frequency response curves of bimodular laminated sector plates ($0^0/90^0$, r_2/r_1=5, b_1/h =50, $\phi = 60^0$, $\varsigma_1 = 0.01$). **74**

Fig. 4.3 Deformed configuration corresponding to positive/negative peaks in the frequency response curves of Fig. 4.2 (c) ($C_IC_IC_IC_I$, q_0 =30 Pa, E_{2t}/E_{2c} = 0.5). **76**

Fig. 4.4 Frequency spectra of steady state response history and steady state strain energy histories corresponding to the frequency response curves of Fig. 4.2 (b) ($C_IS_MC_IS_M$, E_{2t}/E_{2c} = 0.5). **77**

Fig. 4.5 Steady state normal stress (kPa) histories and their frequency spectra corresponding to peak amplitude in the frequency response curves of Fig. 4.2 (b). **78**

Fig. 4.6 GNL Frequency response curves of $S_MC_IS_MC_I$ bimodular laminated sector plates (r_2/r_1=5, b_1/h = 50, $\phi = 60^0$, $\varsigma_1 = 0.01$, q_0 =10 Pa, E_{2t}/E_{2c} = 0.5). **80**

Fig. 4.7 GNL Frequency response curves, frequency spectra and steady state energy history of bimodular laminated $C_IS_MC_IS_M$ sector plates (r_2/r_1 =5, b_1/h = 50, $\phi = 60^0$, $\varsigma_1 = 0.01$, q_0 =10 Pa, **82**

$E_{2t}/E_{2c} = 0.5$).

Fig. 4.8 Frequency response curves, steady state response history, frequency spectra, phase plane plots and steady state strain energy history of bimodular laminated sector plates ($C_I S_M C_I S_M$, $0^0/90^0$, r_2/r_1 =5, r_1/h = 50, r_1= 0.5 m, q_0 =10 Pa, $\varsigma_1 = 0.01$, $E_{2t}/E_{2c} = 0.5$). **84**

Fig. 4.9 Through the thickness distributions of fiber direction normal strain (ε_{11}) and normal stresses (σ_{11}, σ_{22}) in MPa corresponding to peak points of frequency response curves of Fig. 4.8 (a). **87**

Fig. 4.10 Steady state normal stress (MPa) histories and their frequency spectra corresponding to peak amplitude in the frequency response curves of Fig. 4.8 (a). **89**

Fig. 4.11 Frequency response curves, steady state response history, phase plane plot and frequency spectra of bimodular laminated sector plates ($C_I S_M C_I S_M$, r_2/r_1=5, b_1 /h =50, ϕ = 60^0, q_0 = 0.2 Pa, $\varsigma_1 = 0.01$, Material 1). **91**

Fig. 5.1 Geometry and coordinate system of cylindrical panel. **95**

Fig. 5.2 Frequency response curves of bimodular cross-ply laminated cylindrical panels ($0^0/90^0$, L/b = 1, b/h =50, r/h=500, $\varsigma_1 = 0.01$, Material 3). **99**

Fig. 5.3 Displacement variations corresponding to the positive/negative peak amplitudes in the frequency response curves of Fig. 5.2 (b) ($C_I C_I C_I C_I$, q_0 = 4 kPa, Material 3, E_{2t}/E_{2c} = 2.0). **101**

Fig. 5.4 Fiber direction normal strain/stress (ε_{11}/ σ_{11}) and transverse to fiber direction normal stress (σ_{22}) distributions of bimodular laminated $C_I C_I C_I C_I$ cylindrical panel corresponding to peak amplitudes in frequency response curves of Fig. 5.2 (a). **102**

Fig. 5.5 Steady state stresses (σ_{11} and σ_{22}) in MPa versus time (t/T) plots at the centre of outer and inner surfaces corresponding to peak amplitudes in the frequency response curves of Fig. 5.2 (a) over one time period (T). **104**

Fig. 5.6 Frequency spectra of GNL steady state stresses (σ_{11} and σ_{22}) corresponding to Fig. 5.5. **105**

Fig. 5.7 Frequency response curves of bimodular angle-ply laminated cylindrical panels ($C_IC_IC_IC_I$, L/b = 1, b/h =50, r/h=500, q_0= 2 kPa, $\varsigma_1 = 0.01$, Material 3). **106**

Fig. 5.8 Frequency response curves of bimodular cross-ply laminated $C_MS_MC_MS_M$ and $C_IS_MC_IS_M$ cylindrical panels (b/h =50, $\varsigma_1 = 0.05$, q_0= 15 Pa, Material 1). **108**

Fig. 5.9 Steady state stresses (σ_{11} and σ_{22}) versus time (t/T) plots at the centre of outer and inner surfaces corresponding to peak amplitude points in the frequency response curves (Figs. 5.8 (a) and (b), L/b = 2) over one time period (T). **111**

Fig. 5.10 Phase plane plots and frequency spectra of GNL response of bimodular laminated $C_IS_MC_IS_M$ cylindrical panels (L/b =2, b/h =50, r/h=250, $\varsigma_1 = 0.05$, q_0= 15 Pa, Material 1, ω_F/ω= 0.55). **112**

Fig. 5.11 Frequency response curves of bimodular laminated cylindrical panel with and without initial imperfection ($0^0/90^0$, $C_IC_IC_IC_I$, L/b = 1, b/h =50, r/h=500, q_0= 2 kPa, $\varsigma_1 = 0.01$, Material 3: E_{2t}/E_{2c}= 2.0). **113**

Fig. 6.1 Geometry and coordinate system of a laminated conical panel **117**

Fig. 6.2 Frequency response curves of bimodular laminated conical panels ($0^0/90^0$, L/b_1 = 1, b_1/h =50, r_1/h=500, $\alpha = 15^0$, $\varsigma_1 = 0.01$, Material 3). **120**

Fig. 6.3 Fiber and transverse to fiber direction normal strain/stresses (ε_{11}, σ_{11}, σ_{22}) distributions, phase plane plots and frequency spectra corresponding to peak amplitude points in frequency response curves (Fig. 6.2 (a), q_0= 2 kPa). **122**

Fig. 6.4 Steady state stresses (σ_{11} and σ_{22} in MPa) history at the centre of outer and inner surfaces and their frequency spectra corresponding to peak amplitude points in the frequency response curves (Fig. 6.2 (a), q_0= 2 kPa). **126**

Fig. 6.5 Frequency response curves of bimodular laminated conical panels ($0^0/90^0$, q_0= 2 kPa, L/b_1 = 1, b_1/h =50, r_1/h=250, $\varsigma_1 = 0.01$, $C_IC_IC_IC_I$, Material 3). **129**

Fig. 6.6 Fiber direction normal strain (ε_{11}) distribution corresponding to peak amplitude points in frequency response curves (Fig. 6.5 (a), $\alpha = 15^o$). **130**

Fig. 6.7 Steady state stress (σ_{11} and σ_{22} in MPa) history at the centre of outer and inner surfaces corresponding to peak amplitude points in the frequency response curves (Fig. 6.5 (a), $\alpha = 15^o$). **132**

Fig. 6.8 Displacement variations of bimodular laminated conical panels corresponding to peak amplitude points of Fig. 6.5 (b) (E_{2t}/E_{2c} = 0.5, $\alpha = 45^0$). **132**

Fig. 6.9 Frequency response curves of bimodular laminated conical panels ($S_MS_MS_MS_M$, $(0^0/90^0)_N$, q_0= 20 Pa, b_1/h =50, $\alpha = 15^0$, $\varsigma_1 = 0.01$, Material 2: Polyster-Rubber). **134**

Fig. 6.10 Steady state stress (σ_{11}, σ_{22} in kPa) history and frequency spectra at the centre of outer and inner surfaces corresponding to peak amplitude points in frequency response curves of Fig. 6.8 (b). **136**

List of Tables

Table 2.1 Linear free vibration frequencies (rad/s) of unimodular laminated cylindrical panels. **35**

Table 2.2 Non-dimensional free vibration frequencies $\omega_i r_2^2 \sqrt{\rho h/D_0}$ for isotropic annular sector plates (r_1/r_2= 0.2, ϕ = π/3, α = π/2, μ = 0.3, $D_0 = Eh^3/12(1-\mu^2)$). **36**

Table 2.3 Non-dimensional free vibration frequencies $(\omega_i (r_2 - r_1)^2 \sqrt{\rho/E_2} / h)$ for two-layered ($0^0/90^0$) laminated annular sector plates (r_1/r_2= 0.1, ϕ = π/3, α = π/2, h = 0.2 r_2, E_1/E_2=2, E_2= E_3, G_{12}/E_2 = G_{13} /E_2 = 0.6, G_{23}/E_2= 0.5, ν_{12}= ν_{23}= ν_{13}= 0.25). **36**

Table 2.4 Comparison of nondimensional positive and negative half cycle frequencies ($\omega_i b^2 \sqrt{\rho/E_{2c} h^2}$) of $S_M C_I S_M C_I$ bimodular laminated cylindrical panels (b/h = 10, Material 1). **37**

Table 3.1 Fundamental free vibration frequencies (Hz) of bimodular plates. **40**

Table 3.2 Fundamental free vibration frequencies (Hz) of $C_I C_I C_I C_I$ bimodular plates ($0^0/90^0$, L/b = 2.0, b/h = 100, Material 3). **41**

Table 3.3 Peak amplitude in positive and negative half cycles for $S_M S_M S_M S_M$ bimodular laminated plates corresponding to Fig. 3.2. **45**

Table 4.1 Fundamental free vibration frequencies (Hz) of annular sector plates (r_2/r_1 = 5). **72**

Table 5.1 Linear free vibration frequencies (in Hz) of first mode of **96**

cylindrical panels of Material 3 (L/b = 1, b/h =50, r/h = 500)

Table 5.2 Linear free vibration frequencies (in Hz) of first mode of cylindrical panels of Material 1 (b/h = 50) **97**

Table 6.1 Linear free vibration frequencies (in Hz) of fundamental mode of bimodular conical panels of Material 3 ($0^0/90^0$, L/b_1 = 1, b_1/h =50, b_1= 0.5 m). **118**

Table 6.2 Linear free vibration frequencies (in Hz) of fundamental mode of bimodular conical panels of Material 2 ($S_MS_MS_MS_M$, b_1/h =50, $\alpha = 15^0$, b_1= 0.5 m). **118**

Nomenclature and Abbreviations

English Notations

A_{ij} : Extensional stiffness coefficients

b : Width

b_1 : Small end circumferential length of conical panel

B_{ij} : Bending-stretching coupling stiffness coefficients

$\tilde{\mathbf{B}}$: Strain-displacement matrix

C : Damping matrix

D_{ij} : Bending stiffness coefficients

E : Young’s modulus

F : Load vector

G : Shear modulus

h : Total thickness

h_k : k^{th} layer thickness

$\mathbf{H}$: Interpolation matrix

$\mathbf{H_w}$: Interpolation matrix for transverse displacement

k : Layer number

$\mathbf{K}, \mathbf{K_1}, \mathbf{K_2}$: Stiffness matrices

$\mathbf{K_P}$: Positive half cycle linear stiffness matrix

$\mathbf{K_n}$: Negative half cycle linear stiffness matrix

$\mathbf{K_T}$: Tangent stiffness matrix

L : Slant length

$\mathbf{M}$: Mass matrix

N : Number of layers

N_i^0 : Original shape functions

$\bar{N}_i$: Smoothed shape functions

q : Distributed transverse load

$\bar{\mathbf{Q}}_{\mathbf{k}}$: Transformed reduced stiffness coefficients matrix

r : Radius of parallel circle

$r_1,\ r_2$: Small and large end radii of conical panel

t : time

T : Time period

$\tilde{T}$: Kinetic energy

U : Potential energy

U_L, U_Q, U_C : Strain energy contributions from linear, quadratic and cubic restoring forces

u, v, w : Displacements in meridional, circumferential and thickness directions

x, y, z : Meridional, circumferential and radial/thickness coordinates

z_k, z_{k+1} : z coordinates of bottom and top surfaces of the k^{th} layer

Greek Notations

α : Semi-cone angle

$\hat{\alpha}$: Mass proportional damping parameter

β : Stiffness proportional damping parameter

$\boldsymbol{\delta}, \dot{\boldsymbol{\delta}}, \ddot{\boldsymbol{\delta}}$: Displacement, velocity and acceleration vectors

$\boldsymbol{\eta}$: State vector

$\boldsymbol{\varepsilon}_{\mathbf{P}}^{\mathbf{L}}, \boldsymbol{\varepsilon}_{\mathbf{b}}, \boldsymbol{\varepsilon}_{\mathbf{s}}$: Linear mid-surface membrane, bending, transverse shear strain vectors

$\boldsymbol{\varepsilon}_{\mathrm{P}}^{\mathbf{NL}}$: Nonlinear mid-surface membrane strain vector

ε_{11} : Fiber direction normal strain

ϕ : Included angle

ν : Poisson’s ratio

ω : Linear free vibration frequency $= 2\left(\frac{1}{\omega_{\mathrm{ip}}}+\frac{1}{\omega_{\mathrm{in}}}\right)^{-1}$

ω_{ip} : Positive half cycle free vibration frequency of *i*th mode

ω_{in} : Negative half cycle free vibration frequency of *i*th mode

ω_{F} : Forcing frequency

ρ : Density

$\boldsymbol{\sigma}$: Stress

σ_{11} : Fiber direction normal stress

σ_{22} : Transverse to fiber direction normal stress

$\sigma_{11i}, \sigma_{22i}$: Harmonic components of fiber and transverse to fiber direction normal stresses

τ : Shear Stress

θ_x, θ_y : Independent rotations of the meridional and hoop sections

ξ_i : *i*th mode damping factor

Subscripts

0 : Mid surface

1 : Fiber direction

2, 3 : Transverse to fiber directions

t : Tension

c : Compression

Abbreviations

FEM : Finite Element Method

FSDT : First Order Shear Deformation Theory

GL : Geometrically linear

GNL : Geometrically nonlinear

$\mathrm{C_I}$: Immovable clamped

$\mathrm{C_M}$: Movable clamped

F : Free

$\mathrm{S_I}$: Immovable simply supported

$\mathrm{S_M}$: Movable simply supported