

**STRUCTURE OF CERTAIN NONASSOCIATIVE RINGS
WITH COMMUTATORS IN THE NICLEI**

By

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DEPARTMENT OF MATHEMATICS

Submitted

in fulfilment of the requirements
for the award of the degree of
DOCTOR OF PHILOSOPHY

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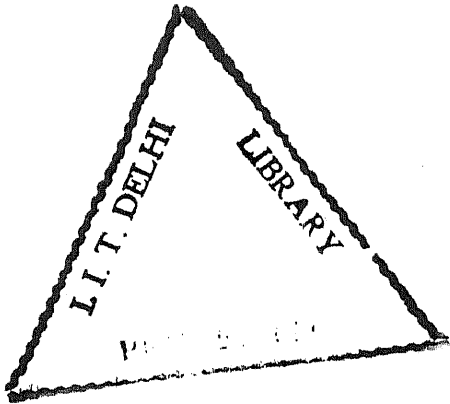


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To

My Parents

CERTIFICATE

This is to certify that the thesis entitled **STRUCTURE OF CERTAIN NONASSOCIATIVE RINGS WITH COMMUTATORS IN THE NUCLEI** which is being submitted by Mr. Dhabalendu Samanta to the **INDIAN INSTITUTE OF TECHNOLOGY, DELHI** for the award of the degree of **DOCTOR OF PHILOSOPHY** in Mathematics is a record of bonafide research work carried out by him under my guidance and supervision.

The thesis has reached the standard of fulfilling the requirements for submission. The results obtained in this thesis have not been submitted in part or full, to any other university or institution for the award of any other degree or diploma.



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ABSTRACT

Our main aim in the thesis is to attempt and solve some problems in certain nonassociative rings with commutators in the nuclei. In chapter I, we have given a brief and up-to-date survey of known results relevant to our works in the thesis. In chapter II, we obtain some results on right ideals of flexible rings with commutators in the nucleus. We have also discussed Peirce decomposition of flexible rings with commutators in the nucleus. Chapter III deals with various kind of Novikov rings. Semiprime weakly Novikov rings with commutators in the right nucleus is shown to be associative. In a semiprime weakly Novikov ring satisfying $(x, y, z) = (x, z, y)$ it is proved that the commutative center is contained in the nucleus. Chapter IV deals with semialternative rings with commutators in the nucleus. We have derived a condition, which gives an orthogonal Peirce decomposition of a semialternative ring in two submodules instead of four submodules. In chapter V, we have studied third power associative antiflexible rings satisfying $[x, (x, y, z)] = 0$. A primitive ring is shown to be associative. Also an attempt has been made to study prime rings. Finally, we have given few examples showing various properties and recorded few problems which arose during our work.

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