

**COMPUTATIONALLY EFFICIENT FINITE
ELEMENT FORMULATION FOR BLOOD FLOW
ANALYSIS IN DEFORMABLE BLOOD VESSELS**

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INDIAN INSTITUTE OF TECHNOLOGY DELHI
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by

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Department of Applied Mechanics

Submitted

in fulfilment of the requirements of the degree of Doctor of Philosophy

to the



**INDIAN INSTITUTE OF TECHNOLOGY DELHI
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- *Dedicated to my parents-*

CERTIFICATE

This is to certify that the thesis entitled ”**Computationally efficient finite element formulation for blood flow analysis in deformable blood vessels**” being submitted by **Mr. Md. Hasan** to the Indian Institute of Technology Delhi for the award of degree of **Doctor of Philosophy** in Applied Mechanics is a record of original, bonafide record of research work carried out by him under our supervision and guidance. The thesis work, in our opinion, has reached the requisite standard fulfilling the requirements for the degree of Doctor of Philosophy.

The results contained in this thesis have not been submitted in part or in full, to any other university or institute for the award of any degree or diploma.

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Abstract

One-dimensional models have been extensively used to study the arterial hemodynamics due to their relative ease of application to a larger arterial network and ability to supply more accurate boundary conditions to the higher dimensional models. These models require a priori assumed axial velocity profile function across the cross section. The comparison of flow characteristics predicted using different a priori assumed or evolving cross-sectional velocity functions has not been reported in the literature. In the present thesis, such a study is undertaken with the main contributions as investigation on the influence of three different a priori assumed cross-sectional velocity profiles on the blood flow characteristics, development of a computationally efficient finite element formulation to eliminate the requirement of a priori assumed cross-sectional velocity profile, formulation of a wave propagation model for blood flow simulation considering a more realistic multi-layered arterial wall wherein each layer is modeled using a viscoelastic material model and investigation on the blood flow characteristics considering blood

as a Non-Newtonian fluid.

Partial differential equations, mathematically representing the physical phenomena of blood flow in the compliant blood vessels, are discretized in the spatial domain by finite element method and in time domain by implicit Crank-Nicolson or Galerkin time integration technique. For the one-dimensional model with a priori assumed axial velocity profile, a 3-noded finite element with flow rate (q), pressure (p) and cross-sectional area (A) as nodal degrees of freedom is used for the discretization along the arterial axis. For the computationally efficient axisymmetric formulation, a 9-noded element is used for the interpolation of axial velocity (u) along length and radius of the artery. The pressure (p) and arterial wall radius (R) are interpolated using a 3-noded element along the length. The radial velocity is assumed to be linear along the radial direction and related to the arterial wall radial velocity. The arterial wall is modeled as single layer elastic, single layered viscoelastic and two-layered (considering media and adventitia) viscoelastic material. A velocity boundary condition is prescribed at the inlet of a single/arterial network and outlet boundary condition is prescribed in terms of 3 parameters based Windkessel model. The radial velocity of the arterial wall is equal to the radial velocity of fluid at the fluid-solid interface. The axial velocity u of solid and fluid at the interface is taken as zero. The inner radius of the deformable arterial wall is taken as equal to the outer radius of fluid domain and this condition is implemented by updating the radial nodal coordinates of fluid

domain at every iteration within a time step. The fluid pressure at the fluid-solid interface is taken as internal pressure acting on the arterial wall. The flow characteristics and the velocity profile function are found to be in good agreement with the 3-dimensional computational results available in the literature. The axial velocity profile is found to be changing with time and axial coordinate of the artery. The present study finds a phase difference between the shear stress at the wall and the flow rate. The magnitude of shear stress τ_{rz} is significantly smaller than the circumferential normal stress $\sigma_{\theta\theta}$ in the arterial wall. The flow characteristics obtained using the axisymmetric formulation for more realistic multi-layered viscoelastic arterial model differ significantly compared to those predicted using 1D model with a priori assumed velocity profile functions. The blood constitutive models considered predict nearly the same flow rate/pressure variation with time with a little noticeable difference in the flow rate prediction using Newtonian and non-Newtonian blood models at diastolic state. Some difference in the flow rate prediction through the Newtonian and non-Newtonian models is observed at diastolic state.

Keywords: Blood flow, layered arterial structure, Finite element method, Non-linear axisymmetric model, viscoelasticity, Non-Newtonian blood models

सारांश

जैवयांत्रिकी साहित्य में, धमनीय रक्तगतिकी का अध्ययन करने के लिए एक आयामी मॉडल का बड़े पैमाने पर उपयोग किया गया है क्योंकि एक आयामी मॉडल आसानी से एक बड़े धमनी नेटवर्क के लिए उपयोग किया जा सकता है तथा उच्च आयामी मॉडल के लिए अधिक सटीक सीमा स्थितियों की आपूर्ति करने की क्षमता भी रखता है इन मॉडलों में, पहले से कल्पित अक्षीय वेग के फलन की आवश्यकता होती है। साहित्य में प्रवाह विशेषताओं का तुलनात्मक अध्ययन अलग-अलग पहले से कल्पित या उभरते अक्षीय वेग फलन का उपयोग करते हुये नहीं किया गया है। प्रस्तुत शोध प्रबंध में एक ऐसा ही अध्ययन किया गया है जिसमें मुख्य योगदान के रूप में तीन अलग-अलग पहले से कल्पित अक्षीय वेग फलन का प्रभाव रक्त प्रवाह की विशेषताओं में, पहले से कल्पित अक्षीय वेग फलन की आवश्यकता समाप्त करने हेतु एक कम्प्यूटेशनल रूप से कुशल नियत परिमित अंश पद्धति का विकास, रक्त प्रवाह सिमुलेशन के लिए अधिक यथार्थवादी बहु परतीय श्यानताप्रत्यास्थ धमनी की दीवार का प्रतिरूप तथा रक्त को अन्यूटनी द्रव के रूप में मानते हुए रक्त प्रवाह विशेषताओं का जांच शामिल है।

आंशिक अवकल समीकरण, गणितीय रूप से अनुवर्ती रक्त वाहिकाओं में रक्त प्रवाह की भौतिकीय घटना का प्रतिरूपक, को स्पेशियल क्षेत्र में परिमित अंश पद्धति द्वारा और समय क्षेत्र में अंतर्निहित क्रैंक-निकोलसन या गैलेरकिन समय समाकलन तकनीक का प्रयोग करते हुए असंततकरण किया गया है। पहले से कल्पित अक्षीय वेग फलन वाले एक-आयामी मॉडल में 3-नोड परिमित अंश का प्रयोग धमनी की लंबाई के असंततकरण के लिए किया गया है जिसके प्रत्येक नोड पर प्रवाह दर (q), दाब (p) और अनुप्रस्थ क्षेत्रफल (A) डिग्री ऑफ फ्रीडम हैं। कम्प्यूटेशनल रूप से कुशल अक्ष सममित निरूपण में, अक्षीय वेग (u) के प्रक्षेप के लिए धमनी की लंबाई और त्रिज्या की दिशा में एक 9-नोड सीमित अंश, तथा दबाव (p) और धमनी दीवार त्रिज्या (R) के प्रक्षेप के लिए धमनी की लंबाई की दिशा में एक 3-नोड सीमित अंश का उपयोग किया गया है। त्रिज्य वेग, जो धमनी की दीवार की त्रिज्य वेग से संबंधित है, को त्रिज्या की दिशा में रेखिक माना गया है। धमनी की दीवार को एकल प्रत्यास्थ परत/एकल श्यानताप्रत्यास्थ परत/ दो (मध्य तथा बाह्य) श्यानताप्रत्यास्थ परतों वाले बेलन की तरह माना गया है। धमनी तंत्र के अंतर्गमन में वेग परिसीमा प्रतिबंध और बहिर्गमन में, तीन राशियों पर आधारित विंडकेसल मॉडल प्रतिबंध का प्रयोग किया गया है धमनी की दीवार का त्रिज्यीय वेग द्रव-ठोस अंतरपृष्ठ में द्रव के त्रिज्यीय वेग के बराबर लिया गया है अंतरपृष्ठ पर ठोस और द्रव का अक्षीय वेग u शून्य लिया गया है। विकृत धमनी की दीवार की आंतरिक त्रिज्या को द्रवीय क्षेत्र के बाहरी त्रिज्या के बराबर लिया गया है और इस स्थिति को हर समय के प्रत्येक पुनरावृत्ति में द्रवीय क्षेत्र के त्रिज्यीय नोडल निर्देशांक का नवीनीकरण करके परिपालन किया गया है। द्रव-ठोस अंतरापृष्ठ पर द्रव का दबाव धमनी की दीवार पर लगने वाले आंतरिक दबाव के बराबर लिया गया है। प्रवाह विशेषताएँ और वेग प्रोफाइल रेखाचित्र साहित्य में उपलब्ध त्रि आयामी अभिकलनात्मक परिणामों के साथ अच्छे ताल मेल में पाये गये हैं। अक्षीय वेग रेखाचित्र, समय और धमनी के अक्षीय निर्देशांक के साथ बदलता हुआ पाया गया है। वर्तमान अध्ययन ये दर्शाता है

कि धमनी की दीवार पर अपरूपण प्रतिबल और प्रवाह दर के बीच एक कलांतर है। धमनी की दीवार में अपरूपण प्रतिबल τ_{rz} का परिमाण परिधीय सामान्य प्रतिबल $\sigma_{\theta\theta}$ से काफी कम है। अधिक यथार्थवादी बहु-परतीय श्यानताप्रत्यास्थ धमनी के साथ अक्ष सममित निरूपण द्वारा प्राप्त खून प्रवाह विशेषताओ, पहले से कल्पित अक्षीय वेग फलन वाले एक आयामी मॉडल से प्राप्त प्रवाह विशेषताओ से काफी भिन्न पायी गयी हैं। डायस्टोलिक अवस्था में अनुमानित प्रवाह दर में काफी कम अंतर के साथ सभी प्रतिबल-बिकृति दर मॉडल मे समय के साथ लगभग समान प्रवाह दर/दाब का अनुमान प्राप्त हुआ है। न्यूटनी और अन्यूटनी मॉडल के प्रयोग से प्राप्त प्रवाह दर के अनुमान में कुछ अंतर डायस्टोलिक अवस्था में मिला है।

कीवर्ड: रक्त प्रवाह, परतीय धमनी संरचना, परिमित अंश पद्धति, आरेखीय अक्ष सममित मॉडल, श्यानताप्रत्यास्थ, अन्यूटनी रक्त प्रतिरूप

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Nomenclature and Abbreviations

English Notations

Q	Average flow rate
p	Pressure
p_d	Diastolic pressure
p_{out}	Pressure outside arterial wall
A	Lumen cross-sectional area at pressure p
A_d	Lumen cross-sectional area at diastolic pressure p_d
t	time
r, z	Radial and axial co-ordinates
L	Length of undeformed artery
h	Undeformed artery wall thickness
f_z	Axial component of the body force
\mathbf{C}^e	Damping matrix

\mathbf{K}^e	Stiffness matrix
\mathbf{K}'^e	Tangent stiffness matrix
\mathbf{d}^e	Degree of freedom matrix
u	Axial Velocity
v	Radial Velocity
R	Arterial wall radius at pressure p
R_o	Outer arterial wall radius
R_d	Arterial wall radius at diastolic pressure p_d
D	Inner wall diameter at diastolic pressure p_d
E	Young's modulus
E_v	Elastic modulus of the spring belongs to viscous part of the SLS model
$E(t)$	Reduced relaxation moduli
\tilde{R}_1, \tilde{R}_2	Windkessel model resistances
C	Windkessel model capacitance
E_∞	Steady state modulus
N	Number of Maxwell elements
E_n	Young's modulus of n^{th} Maxwell element
h_1 & h_2	Thickness of media and adventitia layers of unde- formed artery

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$N_{\theta\theta}$	Circumferential stress resultant
W	Strain energy density function
G	Shear modulus
J_m	Measure of limiting chain extensibility
C	Right Cauchy-Green tensor
E	Green-Lagrange strain tensor
I	Identity matrix
$N_i(\xi, \eta)$	Shape function for 9 noded 2D element
$M_i(\xi)$	Shape function for 3 noded 1D element
$\dot{\epsilon}_{\theta\theta}$	Strain rate

Greek Notations

α	Womersley parameter
ρ	Blood density
μ	Fluid dynamic viscosity
δ_1	Momentum correction factor
τ_w	Frictional force
β_v	Coefficient of viscosity of the SLS model
τ_ϵ	Strain relaxation time

τ_σ	Stress relaxation time
η_v	Coefficient of viscosity of dashpot
ν	Poisson's ratio
Ω	Spatial domain
ω_i	Weight function
\check{w}_i	Weight function for continuity equation
w_i	Weight function for Axial momentum equation
τ_{rz}	Shear stress
$\sigma_{\theta\theta}$	Circumferential normal stress
σ_{rr}	Radial normal stress
$\epsilon_{\theta\theta}$	Circumferential normal strain
ϵ_{zz}	Longitudinal normal strain
σ	Normal stress
ϵ	Normal strain
β_n	Decay rate of n^{th} Maxwell element
ϵ_θ	Circumferential Green's strain
ϵ_z	Axial Green's strain
η_z	unit vector in axial direction
η_r	unit vector in radial direction
ξ, η	Natural coordinates associated with a finite element

CONTENTS

ν_1	Poison's ratio of media layer
ν_2	Poison's ratio of adventitia layer
Δt	Time increment
$\Delta N_{\theta\theta}$	Increment in circumferential stress resultant over time Δt
$\Delta N_{\theta\theta}^R$	History of the circumferential stress resultant
$\Delta \epsilon_{\theta\theta}$	Increment in the circumferential strain

Abbreviations/Acronyms

FSI	Fluid structure interaction
SLS	Standard linear solid
Avg	Average
Max	Maximum
RMS	Root mean square
Superscripts M	Momentum equation
Superscripts C	Continuity equation

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