ROBUST EVENT-TRIGGERED CONTROL OF UNCERTAIN SYSTEMS

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by

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“Every endeavor is covered by some sort of fault, just as fire is covered by smoke. Therefore one should not give up the work which is born of his nature ··· even if such work is full of fault.”

— **Bhagavad Gita** *(Chapter 18, Verse 48)*
[Translated by **Srila Prabhupada**]

I dedicate this piece of work to my late grandmother, Kumudini. In spite of any formal education, she consistently shared her support, insight, tutelage and inspiration to study.

Certificate

This is to certify that the thesis entitled **"Robust Event-triggered Control of Uncertain Systems"** being submitted by **Mr. Niladri Sekhar Tripathy** to the Department of Electrical Engineering, Indian Institute of Technology Delhi, for the award of the Degree of **Doctor of Philosophy** is the record of the bonafide research work carried out by him under our supervision and guidance. In our opinion, the thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted either in part or in full to any other university or institute for the award of any degree or diploma.

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Niladri Sekhar Tripathy

Abstract

Controlling uncertain networked control systems (NCS) with limited communication among subcomponents is a challenging task. Usually in NCS, multiple physical systems interact with their subcomponents through shared communication resources. Therefore effective utilization of these resources is the primary requirement for accomplish the desired goal of controlled system. This fact motivates several researchers towards aperiodic sensing and control beyond the conventional continuous and periodic scheme. In the recent past, it is shown that aperiodic sampling has more benefits over periodic sampling to reduce the consumption of resources. Nowadays, the event-based sampling impressively exhibits the effective reduction of network bandwidth within the feedback loop. This thesis considers event-triggered scheme as a transmission protocol to negotiate information exchange in resilient control for NCS. In event-triggered control, a new information is exchanged among the cyber components only when truly needed.

Mainly in NCS, the network related constraints like delay in the communication medium, packet drop and single-packet versus multiple-packet transmission primarily affect control performance. Apart from communication constraint in feedback loop, the presence of system uncertainties deteriorates the closed loop performance. Mainly parameter variation, disturbances and unmodelled dynamics are the sources of uncertainties. Broadly, system uncertainties are divided in two classes like matched and mismatched one. In matched system, the uncertainty belongs to the range space of input matrix. This condition does not hold for a mismatched system. The thesis presents a robust control algorithm to regulate the closed loop behavior of NCS in the presence of mismatched uncertainty with limited feedback information. An optimal control approach for robust controller design framework is used to derive the control law. The essential idea of proposed robust control approach is an optimal control input is computed for the nominal system which minimizes a certain cost-functional. The derived optimal input for the nominal system is the robust solution of the original uncertain system. The control law is computed and actuated only when a predefined event condition is satisfied. The ISS based analysis is used to derive the event-triggering condition and stability results.

The main results of this thesis are presented in three parts. Firstly, the Part 1 describes

an event-triggered based robust control algorithm for linear systems. Using the results derived in Part 1, the nonlinear systems are considered for analysis in Part 2. In Part 3, the proposed control algorithm is verified on a class of systems. The conclusion and future scope of research are also reported in this part. The primary contributions of this thesis are:

- An event-triggered based robust control law is designed for both continuous and discrete-time linear system. The system model suffers both from matched and mismatched parametric uncertainties. The control law is derived based on the optimal control results of nominal model. The ISS theory is used to derive the event-triggering condition. An analytical proof and numerical results are provided to show the convergences of system states in spite of parametric uncertainty and limitation of feedback information.

- Using emulation-based approach, an event-triggered control law is proposed for a class of nonlinear system. A nonlinear optimal control law is derived without solving the HJB equation. It has been proved that the HJB equation reduces to a differential Riccati equation for such class of system. Deriving the event-triggering rule and ensuring the finite time convergence of system state is another contribution of this thesis.

- Stabilization of a discrete-time mismatched nonlinear system with periodic feedback is proposed in this thesis. To generate the nonlinear robust control input, a nonlinear optimal control problem is solved using NN implementation. The robust control input ensures the asymptotic convergence of uncertain closed loop system.

- The proposed control algorithm is used to stabilize different class of systems like Euler-Lagrange, Lipschitz nonlinear system with limited feedback information. In this formulation, system nonlinearities are treated as a source of uncertainty. Based on optimal control approach, a linear robust control law is derived to stabilize such nonlinear systems.

सार

उपघटकों के बीच सीमित संचार के साथ अनिश्चित नेटवर्क नियंत्रण प्रणालियों (एनसीएस) को नियंत्रित करना एक चुनौतीपूर्ण कार्य है । आमतौर पर एनसीएस में, एकाधिक भौतिक सिस्टम साझा संचार संसाधनों के माध्यम से अपने उपघटकों के साथ सहभागिता करते हैं । इसलिए इन संसाधनों का प्रभावी उपयोग नियंत्रित प्रणाली के वांछित लक्ष्य को पूरा करने के लिए प्राथमिक आवश्यकता है । यह तथ्य पारंपरिक सतत और आवधिक योजना से परे aperiodic संवेदन और नियंत्रण के प्रति कई शोधकर्ताओं को प्रेरित करता है । हाल ही में, यह दिखाया है कि aperiodic नमूना आवधिक नमूना पर अधिक लाभ के लिए संसाधनों की खपत को कम किया है । आजकल, घटना आधारित नमूना प्रभावशाली प्रतिक्रिया लूप के भीतर नेटवर्क बैंडविड्थ की प्रभावी कमी दर्शाती है । इस थीसिस घटना पर विचार करता है एक संचरण प्रोटोकॉल के रूप में योजना को एनसीएस के लिए लचीला नियंत्रण में सूचना विनिमय बातचीत के लिए ट्रिगर । घटना-ट्रिगर नियंत्रण में, एक नई जानकारी केवल जब वास्तव में जरूरत साइबर घटकों के बीच विमर्श किया है ।

मुख्य रूप से एनसीएस में संचार माध्यमों में देरी जैसे नेटवर्क संबंधित बाधाओं, पैकेट ड्रॉप और एकल-पैकेट बनाम एकाधिक-पैकेट संचार मुख्यतः नियंत्रण प्रदर्शन को प्रभावित करते हैं । प्रतिक्रिया पाश में संचार बाधा के अलावा, सिस्टम अनिश्चितताओं की उपस्थिति बंद लूप प्रदर्शन खराब हो जाती है । मुख्य रूप से पैरामीटर भिन्नता, अशांति और unमॉडलिंग की गतिशीलता अनिश्चितताओं के स्रोत हैं । मोटे तौर पर, प्रणाली अनिश्चितताओं मिलान और बेमेल एक तरह दो वर्गों में विभाजित कर रहे हैं । मिलान प्रणाली में, अनिश्चितता इनपुट मैट्रिक्स के रेंज स्पेस के अंतर्गत आता है । यह शर्त एक बेमेल प्रणाली के लिए पकड़ नहीं है । थीसिस सीमित प्रतिक्रिया जानकारी के साथ बेमेल अनिश्चितता की उपस्थिति में एनसीएस के बंद लूप व्यवहार को विनियमित करने के लिए एक मजबूत नियंत्रण एल्गोरिथ्म प्रस्तुत करता है । मजबूत नियंत्रक डिजाइन ढांचे के लिए एक इष्टतम नियंत्रण दृष्टिकोण नियंत्रण कानून प्राप्त करने के लिए प्रयोग किया जाता है । प्रस्तावित मजबूत नियंत्रण दृष्टिकोण के आवश्यक विचार एक इष्टतम नियंत्रण इनपुट नाममात्र की प्रणाली है जो एक निश्चित लागत-कार्यात्मक को कम करने के लिए गणना की है । नाममात्र की प्रणाली के लिए व्युत्पंन इष्टतम इनपुट मूल अनिश्चित प्रणाली के मजबूत समाधान है । नियंत्रण कानून की गणना और केवल जब एक पूर्वनिर्धारित घटना हालत संतुष्ट है हाथ है । आईएसएस आधारित विश्लेषण के लिए घटना-ट्रिगरिंग हालत और स्थिरता के परिणाम प्राप्त किया जाता है ।

इस थीसिस के मुख्य परिणाम तीन भागों में प्रस्तुत कर रहे हैं । सबसे पहले, भाग 1 का वर्णन एक घटना-रैखिक प्रणालियों के लिए आधारित मजबूत नियंत्रण एल्गोरिथ्म ट्रिगर । भाग 1 में व्युत्पंन परिणामों का उपयोग करना, रैखिक प्रणाली भाग 2 में विश्लेषण के लिए माना जाता है । 3 भाग में, प्रस्तावित नियंत्रण एल्गोरिथ्म प्रणालियों के एक वर्ग पर सत्यापित है । इस भाग में अनुसंधान के निष्कर्ष और भविष्य का दायरा भी बताया गया है । इस थीसिस के प्राथमिक योगदान कर रहे हैं:

- एक घटना ट्रिगर आधारित मजबूत नियंत्रण कानून दोनों सतत और असतत समय रैखिक प्रणाली के लिए बनाया गया है । सिस्टम मॉडल दोनों मिलान और बेमेल पैरामीट्रिक अनिश्चितताओं से ग्रस्त है । नियंत्रण कानून नाममात्र मॉडल के इष्टतम नियंत्रण परिणामों के आधार पर व्युत्पंन है । आईएसएस सिद्धांत के लिए घटना-ट्रिगर हालत व्युत्पंन किया जाता है । एक विश्लेषणात्मक सबूत और संख्यात्मक परिणाम पैरामीट्रिक अनिश्चितता और प्रतिक्रिया जानकारी की सीमा के बावजूद प्रणाली राज्यों के अभिसरण दिखाने के लिए प्रदान की जाती हैं ।
- अनुकरण-आधारित दृष्टिकोण का प्रयोग, एक घटना-ट्रिगर नियंत्रण कानून रैखिक प्रणाली के एक वर्ग के लिए प्रस्तावित है । एक रैखिक इष्टतम नियंत्रण कानून HJB समीकरण को सुलझाने के बिना व्युत्पंन है । यह साबित हो गया है कि HJB समीकरण प्रणालियों के ऐसे वर्ग के लिए एक अंतर Riccati समीकरण को कम कर देता है । घटना-ट्रिगर शासन और प्रणाली राज्य के परिमित समय अभिसरण सुनिश्चित करने के इस थीसिस का एक और योगदान है ।

- आवधिक प्रतिक्रिया के साथ एक असतत समय बेमेल रैखिक प्रणाली के स्थिरीकरण इस थीसिस में प्रस्तावित है । रेखीय मजबूत नियंत्रण इनपुट उत्पंन करने के लिए, एक रेखीय इष्टतम नियंत्रण समस्या एनएन कार्यांवयन का उपयोग कर हल है । मजबूत नियंत्रण इनपुट अनिश्चित बंद लूप प्रणाली के asymptotic अभिसरण सुनिश्चित करता है ।

- प्रस्तावित नियंत्रण एल्गोरिथ्म Euler-Lagrange, सीमित प्रतिक्रिया जानकारी के साथ Lipschitz गैर रेखीय प्रणाली जैसे प्रणालियों के विभिंन वर्ग को स्थिर करने के लिए प्रयोग किया जाता है । इस निर्माण में, सिस्टम रेखीय अनिश्चितता के एक स्रोत के रूप में माना जाता है । इष्टतम नियंत्रण के दृष्टिकोण के आधार पर, एक रैखिक मजबूत नियंत्रण कानून ऐसी रैखिक प्रणालियों को स्थिर करने के लिए व्युत्पंन है ।

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Notations

Symbols and main variables

Symbol and main variable	
x	state of continuous-time system at current time
x_k	state of discrete-time system at k^{th} instant
e	measurement error at current time
A	system matrix
B	input matrix
t_k	event-triggering instant for continuous-time system
k_i	event-triggering instant for discrete-time system
J	cost-functional
V	Lyapunov function
u	stabilizing control input
v	virtual control input
K	stabilizing controller gain
L	virtual controller gain
ΔA, ΔB	uncertain matrices
$A(p_0)$	nominal system matrix
τ	inter-event time

Spaces

Space	
$\mathbb{R}$	real line
$\mathbb{R}^n$	the n dimensional Euclidean real space
$\mathbb{R}^{n\times m}$	is a set of all $(n \times m)$ real matrices.
$\mathbb{R}_0^+$	all possible set of positive real numbers
$\mathbb{I}$	all possible set of non-negative integers

Operators

Operator	
$\|x\|$	Euclidean norm of a vector $x \in \mathbb{R}^n$
$\|X\|$	norm of matrix X
$X \leq 0$	negative semi-definiteness of matrix X
$X > 0$	positive definiteness of matrix X
X^T	transpose of matrix X
X^{-1}	inverse of matrix X
$\wedge$	logical and operation
inf	infimum

Abbreviations

Abbreviation	
CT-ETC	Continuous-time Event-triggered Control
DT-ETC	Discrete-time Event-triggered Control
FT-ETC	Finite-time Event-triggered Control
ETC	Event-triggered Control
HJB	Hamilton-Jacobi-Bellman
DT-HJB	Discrete-time Hamilton-Jacobi-Bellman
GHJB	Generalized Hamilton-Jacobi-Bellman
CT-HJB	Continuous-time Hamilton-Jacobi-Bellman
ET-HJB	Event-triggered Hamilton-Jacobi-Bellman
LQR	Linear Quadratic Regulator
ADP	Approximate Dynamic Programming
NN	Neural Network
CPS	Cyber-Physical System
NCS	Networked Control Systems
ZOH	Zero-Order Hold
ISS	Input-to-State Stability
SDRE	State Dependent Riccati Equation
SDC	State Dependent Coefficient
PDE	Partial Differential Equation
DLE	Differential Lyapunov Equation