

**POSTSELECTION-FREE, ENTANGLED BIPHOTON
STATES FROM INTEGRATED WAVEGUIDE
DEVICES IN POTASSIUM TITANYL PHOSPHATE**

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**DEPARTMENT OF PHYSICS
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STATES FROM INTEGRATED WAVEGUIDE
DEVICES IN POTASSIUM TITANYL PHOSPHATE**

by

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Department of Physics

Submitted

in fulfilment of the requirements of the degree of Doctor of Philosophy

to the



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To
All My Loving
Family Members

Certificate

This is to certify that this thesis entitled *Postselection-free, Entangled Biphoton States from Integrated Waveguide Devices in Potassium Titanyl Phosphate*, being submitted by **Vineet Kumar Shukla** to the Indian Institute of Technology Delhi, for the award of the Degree of Doctor of Philosophy, is a record of the original research work carried out by him under the supervision and guidance of Prof. Joyee Ghosh.

I also declare that this work has not been formed the basis of award of any Degree, Diploma, Fellowship, Associateship, or similar title of any University or Institution.

May 2022

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Thank you everybody

IIT DELHI

VINEET KUMAR SHUKLA

Abstract

This thesis mainly focuses on the study of entangled and hyperentangled photon pairs generated in guided wave physical systems, for imminent applications in the fields of nonlinear and quantum optics, quantum information and quantum communication. These applications emphasised the important role played by integrated waveguide devices in compact quantum photonic scalable circuits for generating quantum state multiparticle entanglement. In this context, we have studied and proposed waveguide devices of different designs to efficiently generate photon pairs that could be entangled in one or more degrees of freedom. For this, we numerically and theoretically explored various characteristics of periodically poled potassium titanyl phosphate (ppKTP) and lithium niobate (ppLN) waveguides. The technique we have used, for postselection free generation of entangled photon pairs, is the different biperiodic polings that is capable of satisfying different quasi-phase-matching (QPM) conditions through spontaneous parametric down conversion (SPDC) processes.

In particular, this thesis deals with the generation of different polarization entangled, spatial mode entangled and hyper-entangled photon pairs. For the generation of different polarization entangled biphoton states in $\chi^{(2)}$ based ppKTP and ppLN waveguides, we have studied three examples of different pump wavelengths, with the signal photon being emitted at the telecommunication wavelength of 1550 nm in each case. Polarization-entangled photon pairs are produced in the $|\psi^+\rangle$, or $|\phi^+\rangle$ state, utilizing dual quasi-phase-matching conditions in each case. Both these cases are analyzed separately and compared for KTP and LN. We also compare the efficient generation of maximally entangled photon pairs by calculating the von Neumann entropy in these cases. An analysis of the joint spectral amplitude (JSA) ensures which parameters are ambient for generating frequency-correlated photons in the phase-matching spectrum of each material.

Further, we study the generation of all four polarization-entangled Bell states in a compact waveguide device with a single input pump beam. This is achieved through non-degenerate spontaneous parametric downconversion and quasi-phase-matching with special non-uniform poling in different lengths. These biphoton states consist of the signal again being emitted in a telecommunication wavelength. Through their joint spectral amplitude, we study

the frequency correlation properties of the photon pairs. Polarization-entangled photon pairs are used as a resource of entanglement in various quantum optics experiments and there is a demand for generating all four photonic Bell states in a single waveguide-based device in quantum information applications. The point of interest of this work is that for the first time, we have discussed the possibility of generation of four different polarization-entangled states that resemble the respective Bell states (in case of maximal entanglement) in a single integrated waveguide device, applicable in quantum teleportation, superdense coding and quantum computing. Moreover, such schemes are of considerable interest in applications such as hybrid quantum networks that require nondegenerate and polarization-entangled photon pairs with one photon at a typical wavelength of 1550-nm, suitable for propagation through an optical fiber, and the other at an appropriate wavelength suitable for interaction with an atomic system or a quantum dot.

We also study the generation of four possible photonic hyperentangled states in a potassium titanyl phosphate waveguide device that are postselection free. The device geometry and design enables hyperentanglement in the spatial modal and polarization bases, through multi-period quasi-phase-matching involved in different spontaneous parametric downconversion processes. The spatially entangled part of these hyperentangled states are rendered swappable with the help of two embedded electro-optic phase modulators. Further, a two-path design provides the flexibility of such a device in the choice of polarization-entangled states and is of importance from an experimental point of view. These states are simultaneously entangled in polarization and spatial modes. Further, the inclusion of electro-optic phase modulators in the device provides a choice of entangled state in the spatial modal basis, without affecting polarization entanglement in both cases. On the other hand, a two-path design of the device offers a choice of polarization-entangled states (through selection of Path-1 or Path-2 to collect the biphotons) without affecting the modal entanglement. Such flexible devices in terms of operation and functionality are of great importance from an experimental point of view. The same is also lucrative as the modal basis is not so easy to manipulate while the polarization basis has been exploited at its best. Through the biphoton joint spectral amplitude, we decipher the parameters that make frequency correlation and further entanglement of the photon pairs feasible. Along with spatial mode and polarization, the biphotons also possess correlations in frequency. Our analysis of the joint spectral intensity ensures that the device parameters are ambient for generating frequency-correlated photons throughout the phase-matching spectra.

It further confirms the generation of hyperentangled photon pairs, depicted through their overlapping JSIs.

Such swappable, hyperentangled and postselection-free photon pairs are useful in several applications of quantum communication and information. We also believe such a waveguide device enables us to design efficient quantum circuits to be used in quantum information technology on a chip.

सार

यह थीसिस मुख्य रूप से नॉनलाइनियर और क्वांटम ऑप्टिक्स, क्वांटम सूचना और क्वांटम संचार के क्षेत्र में आसन्न अनुप्रयोगों के लिए निर्देशित तरंग भौतिक प्रणालियों में उत्पन्न उलझे हुए और हाइपरटेन्गल्ड फोटॉन जोड़े के अध्ययन पर केंद्रित है। इन अनुप्रयोगों ने क्वांटम स्टेट मल्टीपार्टिकल उलझाव उत्पन्न करने के लिए कॉम्पैक्ट क्वांटम फोटोनिक स्केलेबल सर्किट में एकीकृत वेवगाइड उपकरणों द्वारा निर्भाई गई महत्वपूर्ण भूमिका पर जोर दिया। इस संदर्भ में, हमने फोटॉन जोड़े को कुशलतापूर्वक उत्पन्न करने के लिए विभिन्न डिज़ाइनों के वेवगाइड उपकरणों का अध्ययन और प्रस्तावित किया है जो स्वतंत्रता के एक या अधिक डिग्री में उलझ सकते हैं। इसके लिए, हमने संख्यात्मक और सैद्धांतिक रूप से समय-समय पर ध्रुवित पोटेशियम टाइटेनाइल फॉस्फेट (पीपीकेटीपी) और लिथियम नाइओबेट (पीपीएलएन) वेवगाइड की विभिन्न विशेषताओं का पता लगाया। हमने जिस तकनीक का उपयोग किया है, उलझे हुए फोटॉन जोड़े की पोस्ट-सेलेक्शन मुक्त पीढ़ी के लिए, अलग-अलग द्विपक्षीय मतदान हैं जो सहज पैरामीट्रिक डाउन रूपांतरण (एसपीडीसी) प्रक्रियाओं के माध्यम से विभिन्न अर्ध-चरण-मिलान (क्यूपीएम) स्थितियों को संतुष्ट करने में सक्षम हैं।

विशेष रूप से, यह थीसिस विभिन्न ध्रुवीकरण उलझी हुई, स्थानिक मोड उलझी हुई और हाइपर-उलझा हुआ फोटॉन जोड़े की पीढ़ी से संबंधित है। $\chi^{(2)}$ आधारित पीपीकेटीपी और पीपीएलएन वेवगाइड्स में विभिन्न ध्रुवीकरण उलझे हुए द्विफोटॉन राज्यों की पीढ़ी के लिए, हमने विभिन्न पंप तरंग दैर्ध्य के तीन उदाहरणों का अध्ययन किया है, प्रत्येक मामले में 1550 एनएम के दूरसंचार तरंग दैर्ध्य पर सिग्नल फोटॉन उत्सर्जित होने के साथ। ध्रुवीकरण-उलझा हुआ फोटॉन जोड़े $|\psi^+\rangle$, या $|\phi^+\rangle$ राज्य में उत्पादित होते हैं, प्रत्येक मामले में दोहरी अर्ध-चरण-मिलान स्थितियों का उपयोग करते हैं। इन दोनों मामलों का अलग-अलग विश्लेषण किया जाता है और केटीपी और एलएन के लिए तुलना की जाती है। हम इन मामलों में वॉन न्यूमैन एन्ट्रॉपी की गणना करके अधिकतम उलझे हुए फोटॉन जोड़े की कुशल पीढ़ी की तुलना भी करते हैं। संयुक्त वर्णक्रमीय आयाम (जेएसए) का विश्लेषण सुनिश्चित करता है कि प्रत्येक सामग्री के चरण-मिलान स्पेक्ट्रम में आवृत्ति-सहसंबंधित फोटॉन उत्पन्न करने के लिए कौन से पैरामीटर परिवेश हैं।

इसके अलावा, हम एक एकल इनपुट पंप बीम के साथ एक कॉम्पैक्ट वेवगाइड डिवाइस में सभी चार ध्रुवीकरण-उलझी बेल राज्यों की पीढ़ी का अध्ययन करते हैं। यह अलग-अलग लंबाई में विशेष गैर-समान

पोलिंग के साथ गैर-पतित सहज पैरामीट्रिक डाउन-रूपांतरण और अर्ध-चरण-मिलान के माध्यम से प्राप्त किया जाता है। इन द्विफोटॉन राज्यों में एक दूरसंचार तरंग दैर्ध्य में फिर से उत्सर्जित होने वाले संकेत शामिल होते हैं। उनके संयुक्त वर्णक्रमीय आयाम के माध्यम से, हम फोटॉन जोड़े के आवृत्ति सहसंबंध गुणों का अध्ययन करते हैं। ध्रुवीकरण-उलझे हुए फोटॉन जोड़े का उपयोग विभिन्न क्वांटम ऑप्टिक्स प्रयोगों में उलझाव के संसाधन के रूप में किया जाता है और क्वांटम सूचना अनुप्रयोगों में एकल वेवगाइड-आधारित डिवाइस में सभी चार फोटोनिक बेल राज्यों को उत्पन्न करने की मांग है। इस काम की रुचि की बात यह है कि पहली बार, हमने चार अलग-अलग ध्रुवीकरण-उलझन वाले राज्यों के निर्माण की संभावना पर चर्चा की है जो संबंधित बेल राज्यों (अधिकतम उलझाव के मामले में) के समान एकीकृत वेवगाइड डिवाइस में लागू होते हैं। क्वांटम टेलीपोर्टेशन, सुपरडेंस कोडिंग और क्वांटम कंप्यूटिंग। इसके अलावा, ऐसी योजनाएं हाइब्रिड क्वांटम नेटवर्क जैसे अनुप्रयोगों में काफी रुचि रखती हैं, जिन्हें 1550-एनएम के एक विशिष्ट तरंग दैर्ध्य पर एक फोटॉन के साथ नॉनडीजेनरेट और ध्रुवीकरण-उलझा हुआ फोटॉन जोड़े की आवश्यकता होती है, जो एक ऑप्टिकल फाइबर के माध्यम से प्रसार के लिए उपयुक्त है, और दूसरा उपयुक्त पर परमाणु प्रणाली या क्वांटम डॉट के साथ बातचीत के लिए उपयुक्त तरंग दैर्ध्य।

हम एक पोटेशियम टाइटेनाइल फॉस्फेट वेवगाइड डिवाइस में चार संभावित फोटोनिक हाइपरएंटेगल राज्यों की पीढ़ी का भी अध्ययन करते हैं जो पोस्टसेलेक्शन मुक्त हैं। डिवाइस ज्यामिति और डिज़ाइन विभिन्न सहज पैरामीट्रिक डाउन-रूपांतरण प्रक्रियाओं में शामिल बहु-अवधि अर्ध-चरण-मिलान के माध्यम से, स्थानिक मोडल और ध्रुवीकरण आधारों में हाइपरएंटेगलमेंट को सक्षम बनाता है। इन हाइपरएंटेगल राज्यों के स्थानिक रूप से उलझे हुए हिस्से को दो एम्बेडेड इलेक्ट्रो-ऑप्टिक चरण मॉड्यूलैटर की मदद से स्वैपेबल प्रदान किया जाता है। इसके अलावा, एक दो-पथ डिज़ाइन ध्रुवीकरण-उलझा हुआ राज्यों की पसंद में इस तरह के एक उपकरण की लचीलापन प्रदान करता है और प्रयोगात्मक दृष्टिकोण से महत्वपूर्ण है। ये राज्य एक साथ ध्रुवीकरण और स्थानिक मोड में उलझे हुए हैं। इसके अलावा, डिवाइस में इलेक्ट्रो-ऑप्टिक चरण मॉड्यूलैटर को शामिल करने से दोनों मामलों में ध्रुवीकरण उलझाव को प्रभावित किए बिना, स्थानिक मोडल आधार में उलझी हुई स्थिति का विकल्प मिलता है। दूसरी ओर, डिवाइस का एक दो-पथ डिज़ाइन मोडल उलझाव को प्रभावित किए बिना ध्रुवीकरण-उलझा हुआ राज्यों (द्विफोटों को इकट्ठा करने के लिए पथ -1 या पथ -2 के चयन के माध्यम से) का विकल्प प्रदान करता है। प्रायोगिक दृष्टिकोण से संचालन और कार्यक्षमता के मामले में इस तरह के लचीले उपकरणों का बहुत महत्व है। यह भी आकर्षक है क्योंकि मोडल आधार में हेरफेर करना इतना आसान नहीं है जबकि ध्रुवीकरण के आधार का सबसे अच्छा शोषण किया गया है। बायफोटॉन संयुक्त वर्णक्रमीय आयाम के माध्यम से, हम

उन मापदंडों को समझते हैं जो आवृत्ति सहसंबंध और फोटॉन जोड़े के आगे के उलझाव को संभव बनाते हैं। स्थानिक मोड और ध्रुवीकरण के साथ-साथ, द्विफोटों में आवृत्ति में सहसंबंध भी होते हैं। संयुक्त वर्णक्रमीय तीव्रता का हमारा विश्लेषण यह सुनिश्चित करता है कि चरण-मिलान स्पेक्ट्रा में आवृत्ति-सहसंबंधित फोटॉन उत्पन्न करने के लिए डिवाइस पैरामीटर परिवेश हैं। यह आगे उनके अतिव्यापी JSI के माध्यम से दर्शाए गए हाइपरएंटेगल्ड फोटॉन जोड़े की पीढ़ी की पुष्टि करता है।

क्वांटम संचार और सूचना के कई अनुप्रयोगों में इस तरह के स्वैपबल, हाइपरएंटेगल्ड और पोस्टसेलेक्शन-मुक्त फोटॉन जोड़े उपयोगी हैं। हम यह भी मानते हैं कि ऐसा वेवगाइड डिवाइस हमें चिप पर क्वांटम सूचना प्रौद्योगिकी में उपयोग किए जाने वाले कुशल क्वांटम सर्किट को डिजाइन करने में सक्षम बनाता है।

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Chapter 2

Anisotropy and nonlinear optical properties of potassium titanyl phosphate

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Motivation:

Quantum entanglement plays a central role in quantum information applications, such as quantum key distribution, quantum teleportation, superdense coding, quantum networks, quantum computing, etc. In particular, entangled photon states are a popular resource of entanglement used as building blocks in the above applications utilizing Bell's inequalities. A large number of experiments have been performed to investigate the production of entangled states of photons [1-7]. These photons can be generated by nonlinear optical processes like spontaneous parametric down-conversion (SPDC), four-wave mixing (FWM), and from atomic cascades. Among these the most efficient and workhorse technique used spontaneous parametric downconversion (SPDC) utilizing the $\chi^{(2)}$ second-order nonlinearity in crystals to generate such photon pairs. SPDC is a quantum mechanical phenomenon, in which one high-energy photon interacts with the nonlinear medium to split into two photons of lower frequencies. The generated photons have properties and share special quantum correlation's that can lead to entanglement. Entanglement being a unique feature of quantum mechanics, these photons can serve as flying qubits, especially for the transfer of quantum information from one point to another point. Apart from time, frequency and energy correlation, the photon pairs can also share polarization correlations, as they can have the same or orthogonal polarizations. And using such correlations they may be further entangled in same property [8-15]. With such motivation, my work is mainly focused on the generation of entangled photon states in different integrated waveguide devices in nonlinear $\chi^{(2)}$ medium especially KTP based on SPDC.

The probability of generating a photon pair is determined by factors like the properties of the optical material, the wavelength of the pump, and the geometry of the material. This process is subjected to conservation of energy and momentum. Energy conservation can be easily expressed by noting that the total energy of the photon pairs created equals the energy of the pump photon such that $\omega_p = \omega_s + \omega_i$, and momentum conservation, or phase matching condition: $\mathbf{k}_p = \mathbf{k}_s + \mathbf{k}_i$. The phase-matching condition depends on the nonlinear material and geometry of the waveguide. Due to dispersion in a material medium and due to a limitation of crystal length it is not possible to obtain perfect phase matching, but one can circumvent this problem or improve the efficiency of the nonlinear crystal by a compensation through phase-matching techniques such as QPM; which involves periodic poling of the nonlinear crystals [16-23].

The physics and technology of generating and manipulating entangled photons in guided wave nonlinear crystals with appreciable second-order nonlinearities such as potassium titanyl phosphate (KTP) or lithium niobate (LN) have been topics of immense interest. These crystals can also be fabricated in the form of waveguides, which enhance the degree of nonlinear interaction, and hence their efficiency, leading to improved performance compared to bulk counterparts. Further, the superposition and entanglement of quantum states in quantum information science has offered a new modal for the creation of devices that in principle can surpass the performance and limits of devices. Waveguide architecture also plays a crucial role in the generation of photons in desired modes [24-28]. Information can be encoded on a photon by utilizing one or more of its properties such as polarization, orbital angular momentum, spatial mode, frequency, time bin etc.. These photons can be entangled in one or multiple degrees of freedom, the latter being called as hyper-entanglement, leading to photons as flying qubits with a high information capacity [3,29-34].

In this thesis, I have studied the generation of entangled and hyper-entangled photon pairs in possible integrated waveguide architectures proposing different efficient devices for the same. Photons are known for speed and robustness, as they can be easily transported, in both free space and optical fibres, with very little interaction with the environment. This allows photons to carry quantum information over long distances and enables photonic based technologies to operate at room temperature. We also believe such waveguide-based devices enables us to design efficient quantum circuits to be used in quantum information technology on a chip or integrated platform.

The objective of the thesis:

The main objective of the thesis is to generate entangled and hyperentangled biphoton states from integrated waveguide devices based on QPM collinear SPDC. The generated biphotons always consists of a signal photon at the telecom wavelength. In particular, the focus has been a device designs for the generation of all four polarization entangled Bell states. Our objective also includes the comparison of generated entangled states in two popular nonlinear materials of the waveguide. The nonlinear materials are Potassium titanyl phosphate (KTP) and Lithium Niobate (LN). KTP is explored as the main material for this study apart from a comparison with LN. In continuation with that, we explored the interesting case of a device

design to generate swappable hyperentangled states of photons with a single input pump beam. The biphotons in this case are hyperentangled in polarization and modal basis. These entangled and hyperentangled states are characterized through different measures such as the von Neumann entropy (S), fidelity (F) and concurrence (C). We have also discussed the joint spectral amplitude (JSA) and common bandwidth of these correlated photons that are significant from a practical point of view [3].

Structure of the thesis:

The thesis is structured as follows. It consists of seven chapters. **Chapter 1** is an introduction to the thesis. This includes the essential concepts and techniques, characterization parameter, etc. which are required for this thesis. In **Chapter 2**, we describe the anisotropy and nonlinearity of potassium titanyl phosphate which is the main nonlinear material for our study in a waveguide device. In **Chapter 3**, we study the generation of two different polarization-entangled states in a multi-period potassium titanyl phosphate waveguide device. The device biphoton pairs are generated utilizing type-II, type-I, and type-0 SPDC processes in the ppKTP waveguide. In this chapter, we also study the parameters for a lossless Y-splitter, which is the part of device. In **Chapter 4**, we study different comparisons as: the two polarization entangled states generated (i) in different materials like KTP and LN, (ii) from different input pump wavelengths leading to a comparison of states with non-degenerate to nearly degenerate biphotons. We optimize the parameters of the waveguide for the generation of non-degenerate photons pairs at telecom wavelength. To characterize the entanglement, we discuss the von Neumann entropy in all these cases. In **Chapter 5**, we propose and study a compact waveguide-based device to generate all four polarization-entangled states that are close to the Bell states, with multiple periodically-poled in an integrated waveguide device with a single input pump beam. **Chapter 6**, we propose and explore another compact integrated device based on multi period waveguide regions to generate hyperentangled photon pairs, being entangled in both polarization and modal DOF. **Chapter 7** summarizes our research findings presented in the thesis and we present the future scope of our work.