

# **Decentralized Stability Analysis, State Estimation and Control of Interconnected Discrete Time Systems**

by

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*Thesis Submitted*

In fulfilment of the  
requirements of the degree of  
*DOCTOR OF PHILOSOPHY*



to the

**INDIAN INSTITUTE OF TECHNOLOGY, DELHI  
APRIL, 1983**

Dedicated  
to  
my beloved parents

Smt. Seetharavamma  
and  
Sri Venkateswara Rao

"The formulation of a patent statement was a blessing. It gave me the opportunity to think about Physics. Moreover, a practical profession is a salvation for a man of my type: an academic career compels a young man to scientific production, and only strong characters can resist that temptations of superficial analysis".

"Geometry remains a mathematical science because the deduction of theorems from axioms remains a purely logical problem; at the same time it is a physical science insofar as its axioms contain assertions relating to natural objects the validity of which can be proved only by experience".

"All knowledge of reality starts from experience and ends in it".

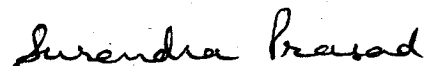
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CERTIFICATE

This is to certify that the thesis entitled, " DECENTRALIZED STABILITY ANALYSIS, STATE ESTIMATION AND CONTROL OF INTERCONNECTED DISCRETE TIME SYSTEMS " being submitted by CH.VENKATA SATYANARAYANA RAO, for the award of Degree of Doctor of Philosophy to the Indian Institute of Technology, Delhi, is a record of bonafide research work he has carried out under our guidance and supervision. The results contained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

  
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### ACKNOWLEDGEMENTS

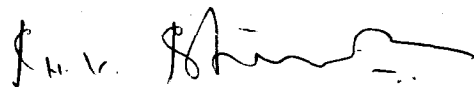
The author would like to take an opportunity to express his indebtedness to his two supervisors Dr. A.K. Mahalanabis, presently Visiting Professor - Lehigh University - Bethlehem - U.S.A. and Dr. Surendra Prasad, Assistant Professor - Indian Institute of Technology - Delhi, for introducing him to the different facets of large scale system studies and for highly stimulating discussions during the entire period of his research career. He also, owes a particular debt of gratitude to Dr. A.K. Sinha - Professor - I.I.T. - Delhi, who made valuable suggestions for presentation of the material. Special appreciation is extended to Dr. P.S. Satsangi - Professor and Head of School of Automation - I.I.T. - Delhi for financial assistance.

He cannot forget to mention his friends, Dr. Joseph Kurian, Dr. Y.B. Reddy, Dr. M. Hanumandlu, Dr. R.G. Rao, Dr. Gosay Das Ray, Dr. S.K. Koul and Dr. V.P. Pyara with whom he has had valuable discussions. Special appreciation is extended to Mr. J.K. Mendiratta, Mr. J. Mahto, Mr. Anand Saxena, Mr. B.M. Gupta, Mrs. Hema Khurana, Mr. R.L. Narasimham, Dr. Vijaya Kumar and Mr. K. Narasimha Murthy for cheerful company

which has made the work easier than expected. He would like to acknowledge particularly, the help received from Mr. P.V.S. Prasad, Mr. P. Venkata Ramana, Mr. Ravindranath, Mr. Subbarao, G. and Mr. Ranganath, V.R.

A very important reason for completion of this thesis was the tremendous moral support received from his brothers, Vijaya Kumar and Bhoo Chandra Sai; sisters, Jansi Lakshmi Bai and Sai Kanya Kumari Devi and friends, Mr. K. Janaki Rama Rao and Mr. G. Seetharam.

Last, but not least, thanks are expressed to Mrs. Sashikala and Mr. P.M. Padhmanaba Nambiar for patient and skilful stencil cutting and Mr. Amar Singh for careful cyclostyling.



CH. V. Satyanarayana Rao

April, 1983

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ABSTRACT

This thesis is concerned with the investigation of some problems of stability analysis, state estimation and control of linear and nonlinear interconnected systems operating in discrete time domain. While most of the earlier works in these areas only treat the continuous time systems, the few papers which consider the discrete time systems, present only centralized solutions. Such solutions though quite useful for multivariable systems, have obvious computational and implementational problems for interconnected systems. Our endeavour here has, therefore, been to develop decentralized forms of solutions to these problems for easier computation and simpler implementation.

The first problem considered is the decentralized stability analysis of linear, discrete, time invariant interconnected deterministic systems. The aim here, is to derive simpler stability criteria for the stability of 'N' interconnected systems. This is done by expressing the Liapunov function for the composite system as a sum of quadratic forms of Liapunov functions for the individual subsystems. It is shown that the interaction effects can be aggregated in a simple manner if suitable bounds are used for the interaction terms. An important result then shown to follow is that there exist N decoupled subsystems whose

stability ensures the stability of the composite system.

Next, an alternative approach to this problem is considered. This requires transformation of the state variable model of a given time invariant deterministic system via the use of its observability properties, to obtain a canonical model such that the system matrix has a block triangular structure. The decentralized stability criteria for the composite system are then established in terms of the block diagonal submatrices of the transformed system.

The Liapunov based aggregation-decomposition techniques developed for the first problem are also shown to be useful for obtaining the stability criteria for a class of nonlinear deterministic systems. It is shown that the composite system of this class can be decomposed into  $N$  decoupled aggregated subsystems with suitable bounds on the interaction terms. A modified form of the Kalman-Szego Lemma is given for decoupled aggregated subsystems. Once again, the stability of the composite system is seen to be related to the decoupled aggregated subsystems. We also suggest an iterative scheme for testing the positive definiteness of the Liapunov matrices of subsystems with interaction, required for the stability criteria.

The Liapunov-based aggregation-decomposition techniques are further shown to be useful for the construction

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of full order asymptotic, decentralized state estimators for interconnected linear systems. The technique permits the design of local gains for the decentralized estimators. A simple algorithm is proposed here for obtaining the elements of these gain vectors.

Finally, we present some new solutions to the problem of construction of decentralized controllers for stabilization of the above class of linear systems. Once again the aggregation-decomposition techniques are employed for the design of local gains for the decentralized state feedback control. A set of conditions for the corresponding decentralized stability criteria is obtained for the composite system stabilization. An iterative scheme is also developed for obtaining the feedback gain matrices.

All the results mentioned here are illustrated through suitably constructed numerical examples.

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