SCATTERING OF MASSLESS DIRAC FERMIONS IN MODULATED GRAPHENE JUNCTIONS AND ITS EFFECT ON ELECTRON TRANSPORT

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Scattering of massless Dirac fermions in modulated graphene junctions and its effect on electron transport

by

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Dedicated to

my parents
and
late grandparents

CERTIFICATE

This is to certify that the thesis entitled "**Scattering of massless Dirac fermions in modulated graphene junctions and its effect on electron transport**" submitted by **Partha Sarathi Banerjee**, for the award of the degree of **Doctor of Philosophy** in **Physics** at **Indian Institute of Technology, Delhi**, is a record of original research carried out by him under our supervision. To the best of our knowledge, the matter embodied in this thesis has not been submitted for the award of any other degree or diploma.

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ABSTRACT

Graphene, with its low-energy quasiparticles described by massless Dirac fermions (MDFs), provides an unparalleled platform to study and manipulate quantum transport phenomena. In condensed matter physics, the role of external electrostatic, magnetic, and periodic potentials in controlling quasiparticle dynamics and generating emergent transport regimes has been extensively studied and established as a key paradigm. This thesis investigates three such problems, establishing theoretical connections to optics, superconductivity, and quantum Hall physics, while highlighting pathways toward graphene-based device applications.

In the first part, we studied quasiparticle scattering from two-dimensional quantum dot lattices using the Lippmann–Schwinger approach in the first Born approximation. We showed that the differential scattering cross-section is governed by the Fourier transform of the potential, drawing a precise analogy to Fraunhofer diffraction in optics. Angle-resolved resistivity revealed clear signatures of lattice configuration, defects, and moiré patterns, suggesting that electronic transport can serve as a probe of structural features. Importantly, this constitutes the first demonstration of fractional Fourier transform behavior in a condensed matter system, thereby extending the scope of optical analogies to electronic scattering.

In the second part, we analyzed superconducting transport in graphene Josephson junctions incorporating one-dimensional electrostatic and magnetic barriers. Within the Dirac – Bogoliubov – de Gennes formalism, we found that such barriers act as effective refractive-index media for MDFs, tuning the strength of Andreev reflection and modulating the Josephson current. This establishes condensed matter analogues of reflection and refraction phenomena, while providing a mechanism for external control of dissipationless current flow in graphene-based superconducting devices.

In the final part, we examined graphene-based superconductor–quantum Hall–superconductor junctions under external electrostatic modulation. Employing a transfer-matrix framework, we

demonstrated that barrier-induced intermediate chiral edge states supply new current-carrying channels and strongly influence Josephson transport. The analysis further underscored the role of interface scattering and revealed how periodic modulation introduces additional tunability of conductivity, emphasizing the rich interplay between superconductivity and quantum Hall physics.

Taken together, these studies demonstrate that engineered external potentials offer a versatile means of controlling MDF scattering and superconducting transport in graphene systems. By elucidating optical analogues of electronic scattering,tuning the Josephson current in magnetically modulated josephson junctions and the emergence of intermediate chiral edge states in superconductor-quantum Hall-superconductor junctions with electrostatic barrier, this thesis advances a unified perspective on graphene transport with implications for electronic imaging, superconducting device design, and quantum information technologies.

सार

ग्राफीन, अपने निम्न-ऊर्जा क्वासिपार्टिकल्स के साथ, जिनका वर्णन द्रव्यमानहीन डिराक फर्मिऑन (एमडीएफ) द्वारा किया जाता है, क्वांटम परिवहन परिघटनाओं के अध्ययन और संचालन के लिए एक अद्वितीय मंच प्रदान करता है। संघनित पदार्थ भौतिकी में, क्वासिपार्टिकल गतिकी को नियंत्रित करने और आकस्मिक परिवहन व्यवस्थाओं को उत्पन्न करने में बाह्य स्थिरवैद्युत, चुंबकीय और आवर्ती विभवों की भूमिका का व्यापक अध्ययन किया गया है और इसे एक प्रमुख प्रतिमान के रूप में स्थापित किया गया है।

यह शोध ऐसी तीन समस्याओं की जाँच करता है, प्रकाशिकी, अतिचालकता और क्वांटम हॉल भौतिकी के साथ सैद्धांतिक संबंध स्थापित करते हुए, ग्रेफीन आधारित उपकरण अनुप्रयोगों की ओर मार्ग प्रशस्त करता है।

पहले भाग में, हमने प्रथम बोर्न सन्निकटन में लिपमैन-श्विंगर दृष्टिकोण का उपयोग करते हुए द्वि-आयामी क्वांटम बिंदु जालकों से क्वासिपार्टिकल प्रकीर्णन का अध्ययन किया। हमने दिखाया कि विभेदक प्रकीर्णन अनुप्रस्थ काट विभव के फूरियर रूपांतरण द्वारा नियंत्रित होता है, जो प्रकाशिकी में फ्रौनहोफर विवर्तन के साथ एक सटीक सादृश्य प्रस्तुत करता है। कोण-समाधान प्रतिरोधकता ने जालक विन्यास, दोषों और मोइरे पैटर्न के स्पष्ट संकेत प्रकट किए, जिससे पता चलता है कि इलेक्ट्रॉनिक परिवहन संरचनात्मक विशेषताओं की जाँच के रूप में कार्य कर सकता है। महत्वपूर्ण रूप से, यह संघनित पदार्थ तंत्र में आंशिक फूरियर रूपांतरण व्यवहार का पहला प्रदर्शन है, जिससे प्रकाशिक सादृश्यों का दायरा इलेक्ट्रॉनिक प्रकीर्णन तक विस्तृत हो जाता है।

दूसरे भाग में, हमने एक आयामी स्थिरवैद्युत और चुंबकीय अवरोधों को समाहित करते हुए ग्राफीन जोसेफसन जंक्शनों में अतिचालक परिवहन का विश्लेषण किया। डिराक-बोगोलिउबोव-डी गेनेस औपचारिकता के अंतर्गत, हमने पाया कि ऐसे अवरोध एमडीएफ के लिए प्रभावी अपवर्तनांक माध्यम के रूप में कार्य करते हैं, एंड्रीव परावर्तन की प्रबलता को समायोजित करते हैं और जोसेफसन धारा को संशोधित करते हैं। यह परावर्तन और अपवर्तन परिघटनाओं के संघनित पदार्थ अनुरूपों को स्थापित करता है, साथ ही ग्राफीन-आधारित अतिचालक उपकरणों में अपव्यय रहित धारा प्रवाह के बाह्य नियंत्रण के लिए एक तंत्र प्रदान करता है।

अंतिम भाग में, हमने बाह्य स्थिरवैद्युत मॉडुलन के अंतर्गत ग्राफीन-आधारित अतिचालक-क्वांटम हॉल-अतिचालक संधियों का परीक्षण किया। स्थानांतरण-मैट्रिक्स ढाँचे का उपयोग करते हुए, हमने प्रदर्शित किया कि अवरोध-प्रेरित मध्यवर्ती किरल किनारा अवस्थाएँ नए धारा-वाहक चैनल प्रदान करती हैं और जोसेफसन परिवहन को दृढ़ता से प्रभावित करती हैं। विश्लेषण ने इंटरफ़ेस प्रकीर्णन की भूमिका को

और रेखांकित किया और बताया कि कैसे आवर्ती मॉडुलन चालकता की अतिरिक्त ट्यूनेबिलिटी प्रस्तुत करता है, जो अतिचालकता और क्वांटम हॉल भौतिकी के बीच समृद्ध अंतर्संबंध पर बल देता है।

कुल मिलाकर, ये अध्ययन प्रदर्शित करते हैं कि अभियांत्रित बाह्य विभव ग्राफीन प्रणालियों में एमडीएफ प्रकीर्णन और अतिचालक परिवहन को नियंत्रित करने का एक बहुमुखी साधन प्रदान करते हैं। इलेक्ट्रॉनिक प्रकीर्णन के प्रकाशिक अनुरूपों को स्पष्ट करके, चुंबकीय रूप से मॉडुलित जोसेफसन संधियों में जोसेफसन धारा को ट्यून करके और स्थिरवैद्युत अवरोध वाले अतिचालक-क्वांटम हॉल-अतिचालक संधियों में मध्यवर्ती किरल किनारा अवस्थाओं के उद्भव द्वारा, यह शोध प्रबंध ग्राफीन परिवहन पर एक एकीकृत दृष्टिकोण प्रस्तुत करता है जिसके इलेक्ट्रॉनिक इमेजिंग, अतिचालक उपकरण डिज़ाइन और क्वांटम सूचना प्रौद्योगिकियों पर प्रभाव पड़ता है।

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