APPROXIMATION ALGORITHMS FOR THE NETWORK DESIGN PROBLEM

by

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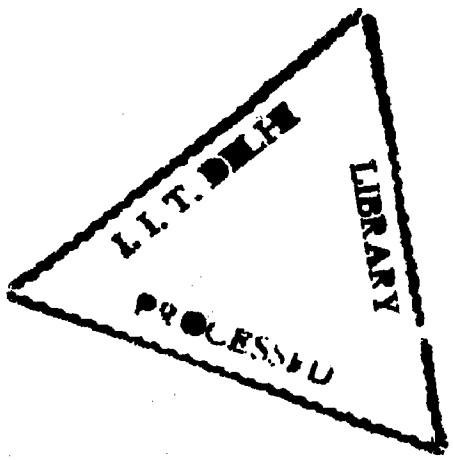

Certificate

This is to certify that the thesis titled "Approximation algorithms for the Network design problem" being submitted by Manica Aggarwal to the Indian Institute of Technology, Delhi, for the award of the Degree of Doctor of Philosophy, is a record of bona-fide research carried out by her under my supervision. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any other degree or diploma.

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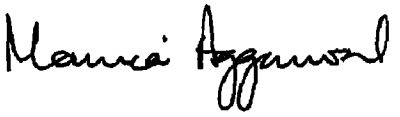

(Manica Aggarwal)

Abstract

This thesis deals with the polynomial time approximability of some of the optimization problems arising in network design, which are NP-complete in their decision version. We focus on the algorithmic aspects of such network design problems. The recent past has witnessed notable interest and work in this field due to the practical applicability of such problems. This thesis aims at further developing the existing concepts and gains insight into the problem and its possible solutions.

The main contributions comprise of the following four subjects.

- We give an approximation algorithm for the uncapacitated Steiner network design problem. Our algorithm gives a solution that in the worst case is at most $2\mathcal{H}(|A| - 1)$ times the optimal solution, where A is the set of sites and $\mathcal{H}(n) = 1 + \frac{1}{2} + \ldots + \frac{1}{n}$, is the n^{th} harmonic sum function. We give a geometric scaling technique that enables us to achieve this factor.

- For the generalized constrained forest problem we give a $2\lceil \log |A| \rceil$ factor approximation algorithm, where A is the set of vertices that have a non-zero requirement.

- We give the first known approximation algorithm for the degree constrained spanning subgraph problem. Our algorithm finds a solution to the problem that in the worst case is 3/2 times the optimal solution. An interesting observation with respect to our algorithm is that it not only gives a performance guarantee of 3/2 but also an absolute guarantee of $n/2$. This observation is used in proving that our algorithm guarantees a solution that is within 4/3 the optimal solution, except for a small set of problems.

- We extend our algorithm for the degree constrained spanning subgraph problem, to the case when the graph has weights on the edges. For the weighted degree constrained spanning subgraph problem the solution our algorithm yields is in the worst case twice the optimal solution.

For all the problems considered in this thesis, we give examples to illustrate that our analysis is tight.

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